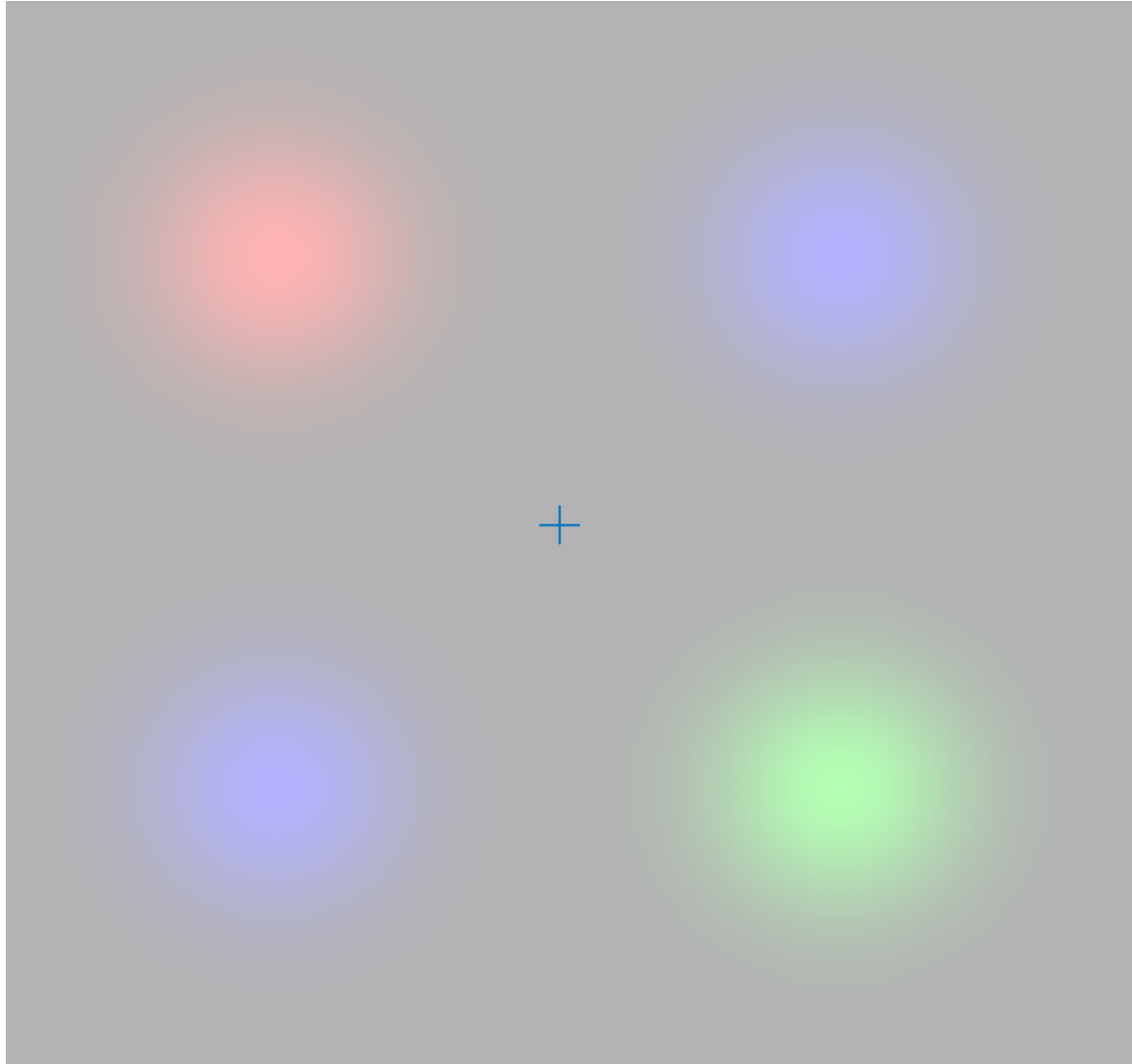


Active Inference

Annual Conference of the Australasian Bayesian
Network Modelling Society





Active Inference

Self-evidencing

Message passing

Discrete time (planning)

Continuous time (movement)

Hierarchical models

Summary



Active Inference

Self-evidencing

Message passing

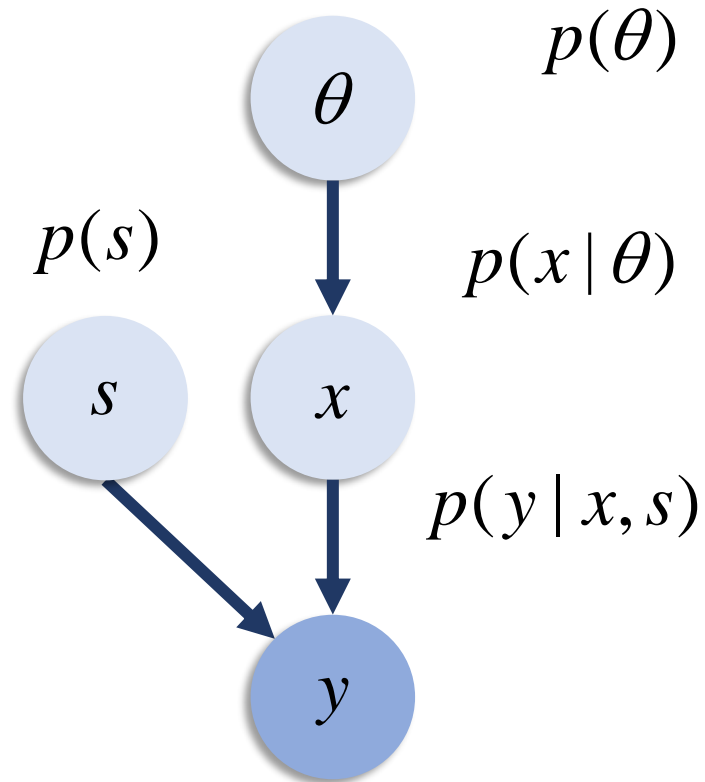
Discrete time (planning)

Continuous time (movement)

Hierarchical models

Summary

Self-evidencing



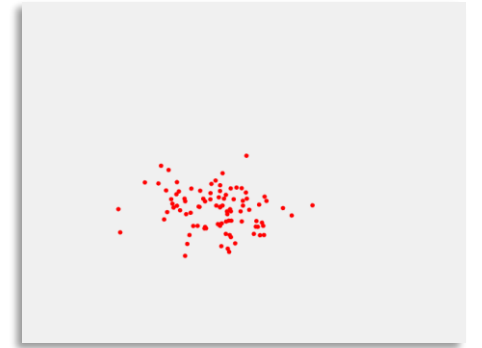
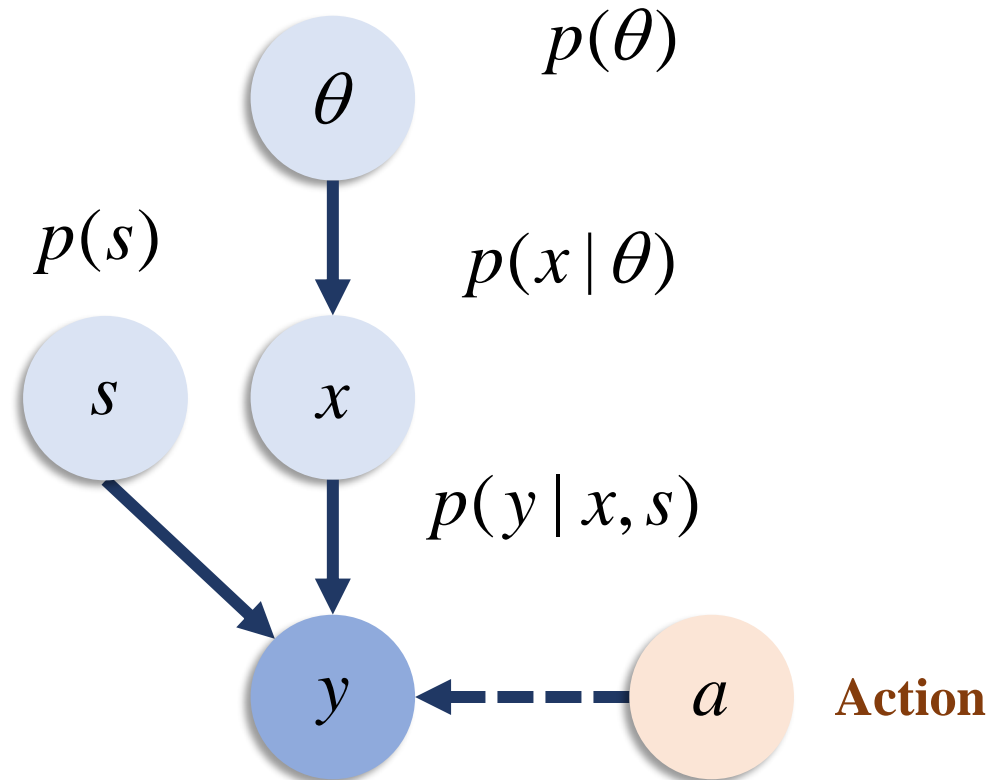
$$p(y) = \sum_s \iint p(y, x, s, \theta) dx d\theta$$

Evidence

$$p(y, x, s, \theta) = p(y | x, s) p(s) p(x | \theta) p(\theta)$$

Model

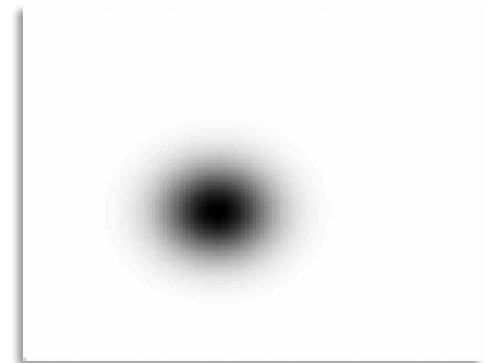
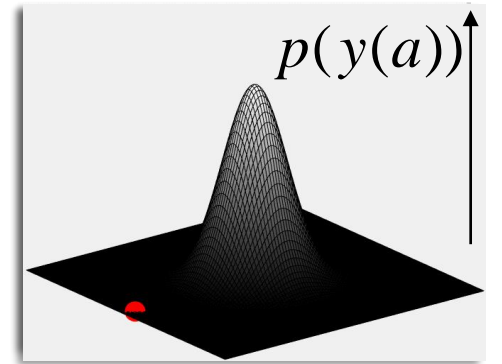
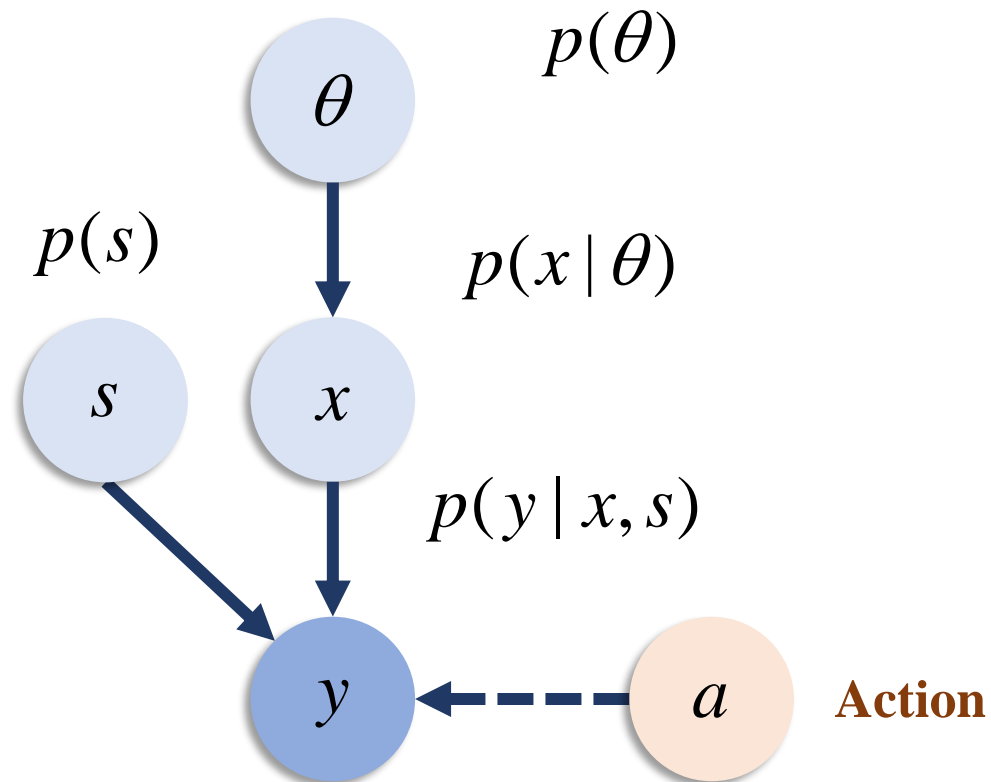
Self-evidencing



$$p(y(a)) = \sum_s \iint p(y(a), x, s, \theta) dx d\theta \quad \text{Evidence}$$

$$p(y(a), x, s, \theta) = p(y(a) | x, s) p(s) p(x | \theta) p(\theta) \quad \text{Model}$$

Self-evidencing



$$p(y(a)) = \sum_s \iint p(y(a), x, s, \theta) dx d\theta$$

Evidence

$$p(y(a), x, s, \theta) = p(y(a) | x, s) p(s) p(x | \theta) p(\theta)$$

Model

Free energy

$$F(y(a)) = \underbrace{E_q [\ln q(x, s, \theta) - \ln p(x, s, \theta | y)]}_{=D_{KL}[q(x, s, \theta) || p(x, s, \theta | y)]} - \ln p(y(a)) \geq 0$$

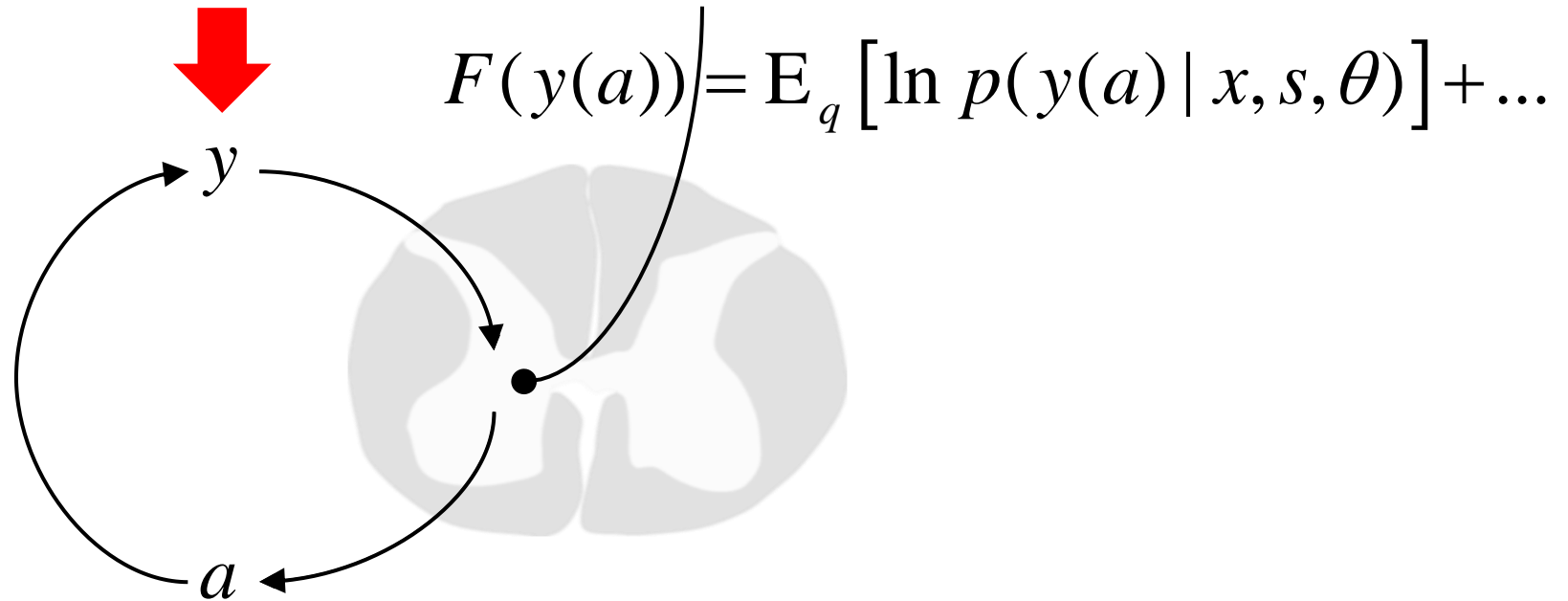
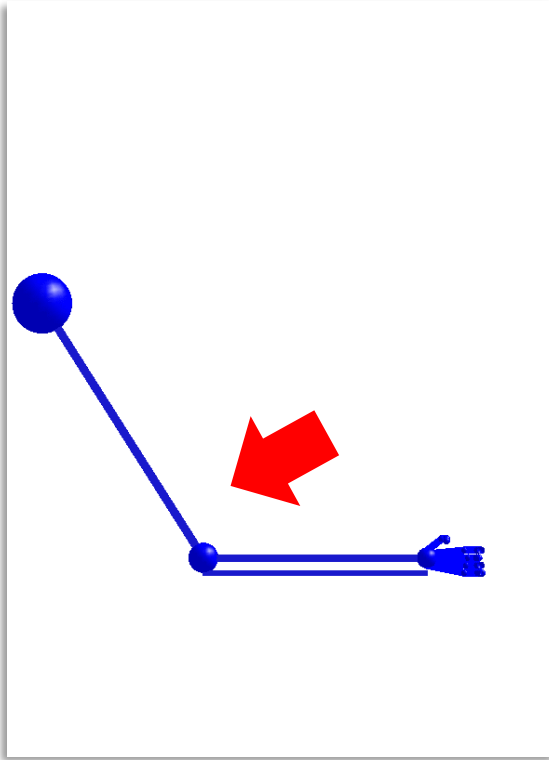
Variational inference

$$q(x, s, \theta) = \arg \min_q F(y(a)) \approx p(x, s, \theta | y)$$
$$\Rightarrow F(y(a)) \approx -\ln p(y(a))$$

Active inference

$$q(x, s, \theta) = \arg \min_q F(y(a)) \approx p(x, s, \theta | y)$$
$$a = \arg \min_a F(y(a)) \approx \arg \max_a p(y(a))$$

Reflexes



Active inference

$$q(x, s, \theta) = \arg \min_q F(y(a)) \approx p(x, s, \theta | y)$$

$$a = \arg \min_a F(y(a)) \approx \arg \max_a p(y(a))$$



Active Inference

Self-evidencing

Message passing

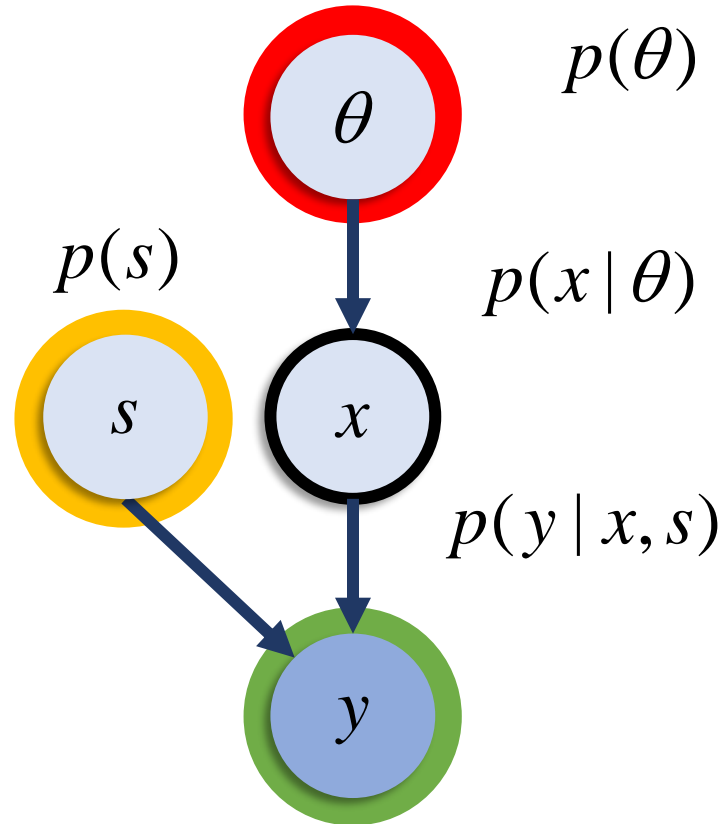
Discrete time (planning)

Sequential time (movement)

Hierarchical models

Summary

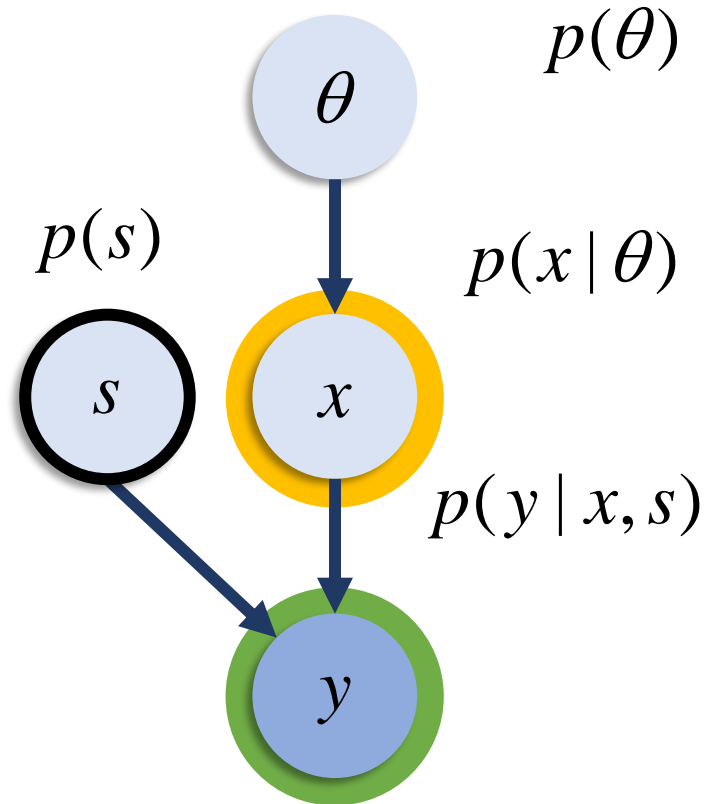
Markov blankets



Parents
Children
Co-parents

$$p(\mu, \eta | b) = p(\mu | b) p(\eta | b) \Leftrightarrow \mu \perp \eta | b$$

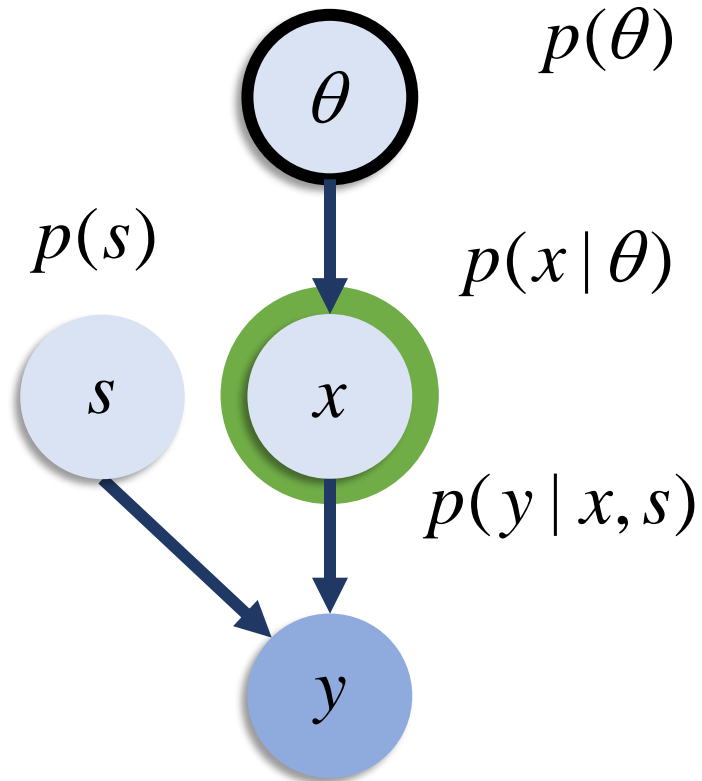
Markov blankets



Parents
Children
Co-parents

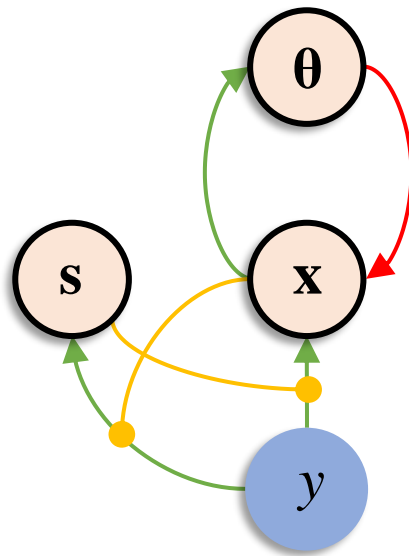
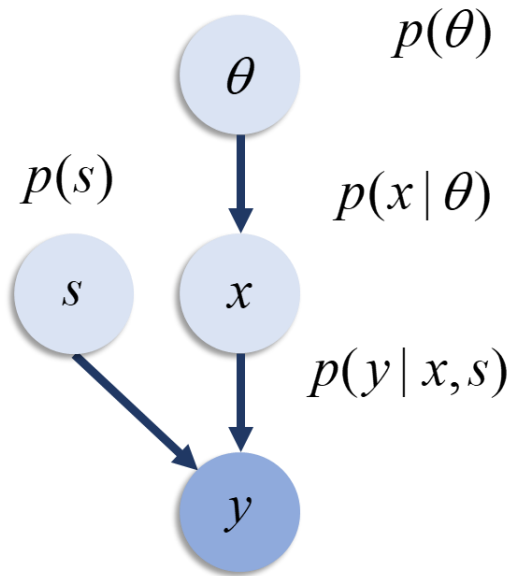
$$p(\mu, \eta | b) = p(\mu | b) p(\eta | b) \Leftrightarrow \mu \perp \eta | b$$

Markov blankets



Parents
Children
Co-parents

$$p(\mu, \eta | b) = p(\mu | b) p(\eta | b) \Leftrightarrow \mu \perp \eta | b$$



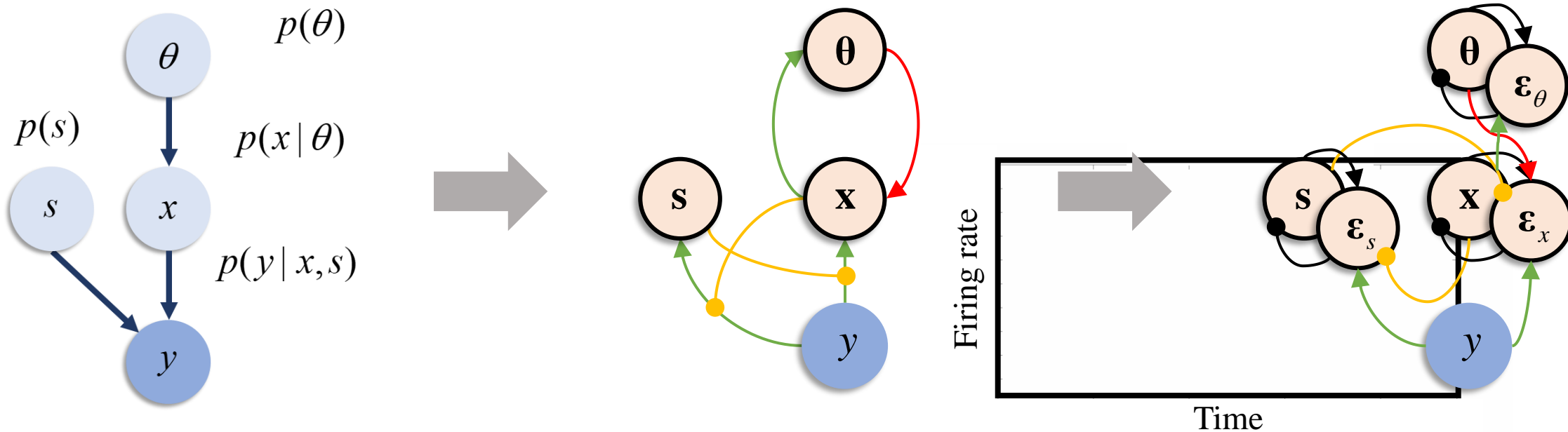
Parents
Children
Co-parents

Variational inference

$$q(\theta) = \arg \min_{q(\theta)} \left\{ \dots \mathbb{E}_{q(\theta)q(x)} [\ln p(x | \theta)] + \mathbb{E}_{q(\theta)} [\ln p(\theta)] - \mathbb{E}_{q(\theta)} [\ln q(\theta)] \right\}$$

$$q(x) = \arg \min_{q(x)} \left\{ \dots \mathbb{E}_{q(\theta)q(x)} [\ln p(x | \theta)] + \mathbb{E}_{q(x)q(s)} [\ln p(y | x, s)] - \mathbb{E}_{q(x)} [\ln q(x)] \right\}$$

$$q(s) = \arg \min_{q(s)} \left\{ \dots \mathbb{E}_{q(x)q(s)} [\ln p(y | x, s)] - \mathbb{E}_{q(s)} [\ln q(s)] \right\}$$



$$\dot{\boldsymbol{\theta}} = \boldsymbol{\varepsilon}_{\theta}$$

Dynamic (gradient descent) inference

$$\boldsymbol{\varepsilon}_{\theta} = \nabla_{\phi(\boldsymbol{\theta})} \mathbb{E}_{q_{\boldsymbol{\theta}}(\boldsymbol{\theta})} [\ln q_{\boldsymbol{\theta}}(\boldsymbol{\theta})] - \nabla_{\phi(\boldsymbol{\theta})} \mathbb{E}_{q_{\boldsymbol{\theta}}(\boldsymbol{\theta}) \underline{q_{\mathbf{x}}(x)}} [\ln p(x | \boldsymbol{\theta})] - \nabla_{\phi(\boldsymbol{\theta})} \mathbb{E}_{q_{\boldsymbol{\theta}}(\boldsymbol{\theta})} [\ln p(\boldsymbol{\theta})]$$

$$\dot{\mathbf{x}} = \boldsymbol{\varepsilon}_{\mathbf{x}}$$

$$\boldsymbol{\varepsilon}_{\mathbf{x}} = \nabla_{\phi(\mathbf{x})} \mathbb{E}_{q_{\mathbf{x}}(x)} [\ln q_{\mathbf{x}}(x)] - \nabla_{\phi(\mathbf{x})} \mathbb{E}_{\underline{q_{\boldsymbol{\theta}}(\boldsymbol{\theta})} q_{\mathbf{x}}(x)} [\ln p(x | \boldsymbol{\theta})] - \nabla_{\phi(\mathbf{x})} \mathbb{E}_{q_{\mathbf{x}}(x) \underline{q_{\mathbf{s}}(s)}} [\ln p(y | x, s)]$$

$$\dot{\mathbf{s}} = \boldsymbol{\varepsilon}_{\mathbf{s}}$$

$$\boldsymbol{\varepsilon}_{\mathbf{s}} = \nabla_{\phi(\mathbf{s})} \mathbb{E}_{q_{\mathbf{s}}(s)} [\ln q_{\mathbf{s}}(s)] - \nabla_{\phi(\mathbf{s})} \mathbb{E}_{\underline{q_{\mathbf{x}}(x)} \underline{q_{\mathbf{s}}(s)}} [\ln p(y | x, s)]$$



Active Inference

Self-evidencing

Message passing

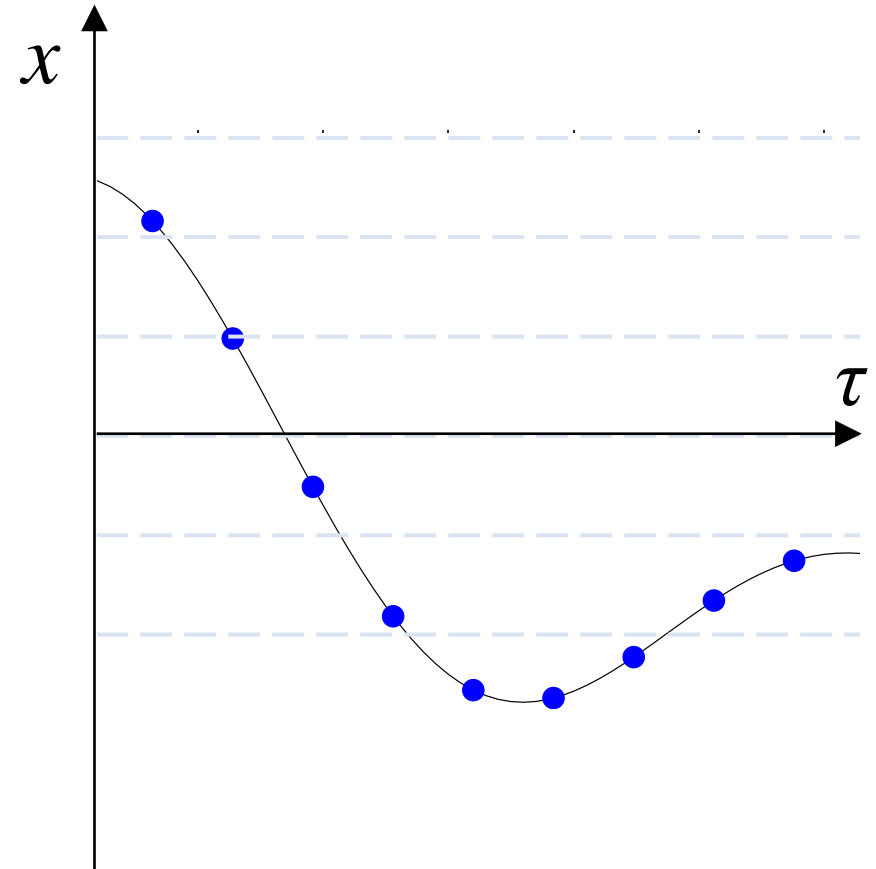
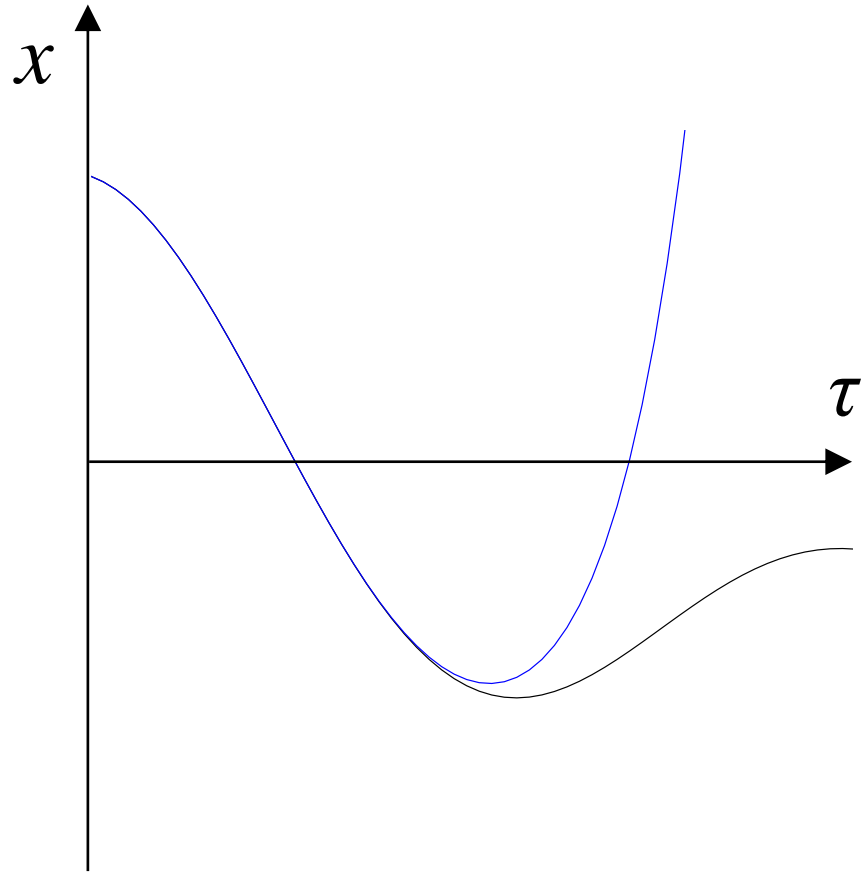
Discrete time (planning)

Continuous time (movement)

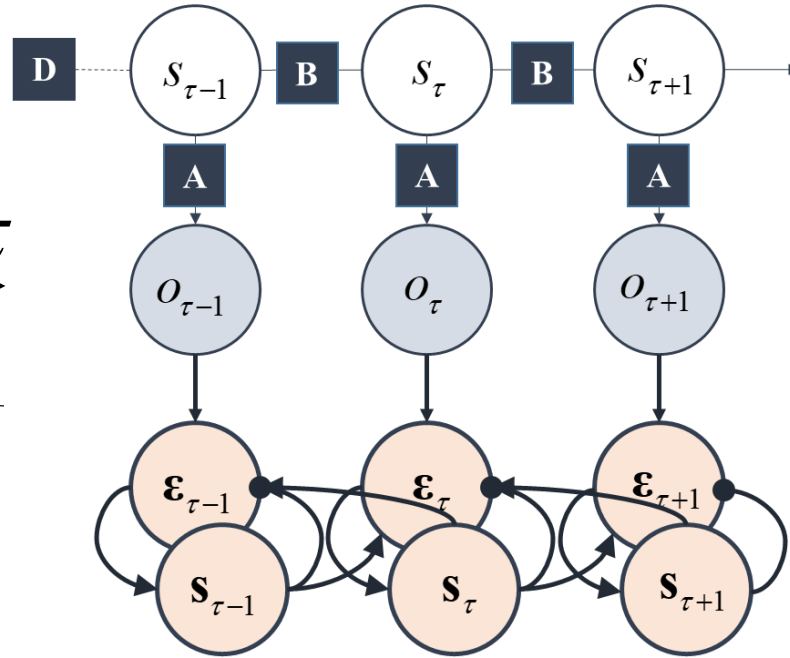
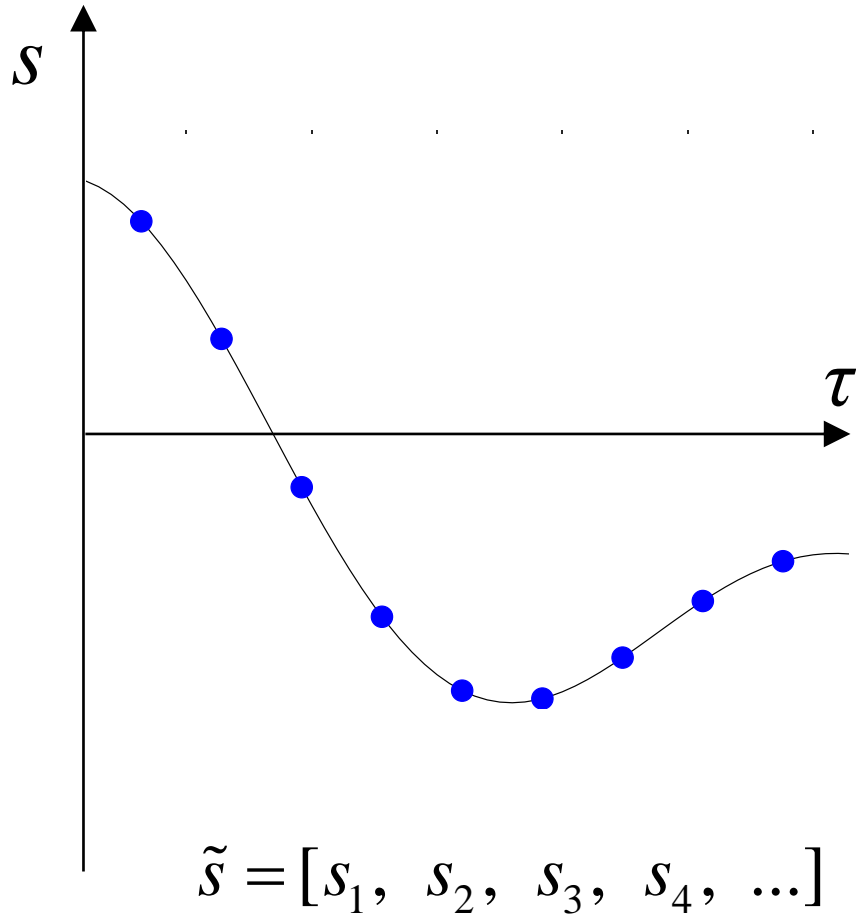
Hierarchical models

Summary

Representing dynamics in generative models



Discrete time models



Generative model

$$P(\tilde{o}, \tilde{s}) = P(s_1) \prod_{\tau} P(s_{\tau+1} | s_{\tau}) P(o_{\tau} | s_{\tau})$$

$$P(o_{\tau} | s_{\tau}) = \text{Cat}(\mathbf{A})$$

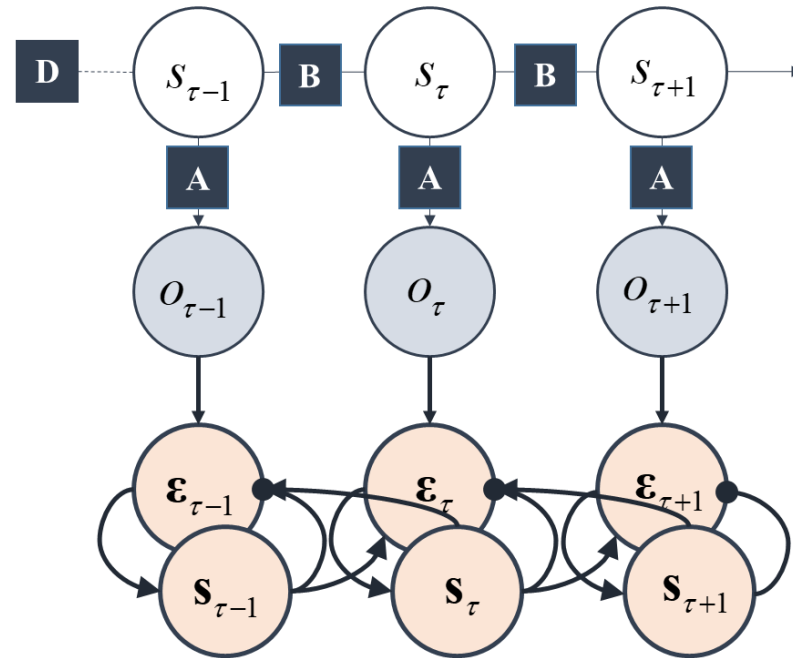
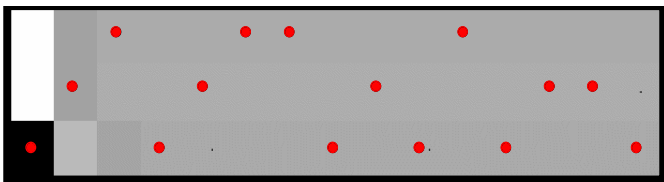
$$P(s_{\tau} | s_{\tau-1}) = \text{Cat}(\mathbf{B})$$

$$P(s_1) = \text{Cat}(\mathbf{D})$$

Bayesian message passing

$$\mathbf{s}_{\tau} = \sigma(\mathbf{v}_{\tau}); \quad \dot{\mathbf{v}}_{\tau} = \boldsymbol{\varepsilon}_{\tau}$$

$$\boldsymbol{\varepsilon}_{\tau} = \ln \mathbf{A} \cdot o_{\tau} + \frac{1}{2} \ln(\mathbf{B}_{\tau} \mathbf{s}_{\tau}) + \frac{1}{2} \ln(\mathbf{B}_{\tau+1}^{\dagger} \mathbf{s}_{\tau+1}) - \ln \mathbf{s}_{\tau}$$



Generative model

$$P(\tilde{o}, \tilde{s}) = P(s_1) \prod_{\tau} P(s_{\tau+1} | s_{\tau}) P(o_{\tau} | s_{\tau})$$

$$P(o_{\tau} | s_{\tau}) = \text{Cat}(\mathbf{A})$$

$$P(s_{\tau} | s_{\tau-1}) = \text{Cat}(\mathbf{B})$$

$$P(s_1) = \text{Cat}(\mathbf{D})$$

Bayesian message passing

$$\mathbf{s}_{\tau} = \sigma(\mathbf{v}_{\tau}); \quad \dot{\mathbf{v}}_{\tau} = \boldsymbol{\epsilon}_{\tau}$$

$$\boldsymbol{\epsilon}_{\tau} = \ln \mathbf{A} \cdot o_{\tau} + \frac{1}{2} \ln(\mathbf{B}_{\tau}^{\dagger} \mathbf{s}_{\tau}) + \frac{1}{2} \ln(\mathbf{B}_{\tau+1}^{\dagger} \mathbf{s}_{\tau+1}) - \ln \mathbf{s}_{\tau}$$

Free energy

$$\begin{aligned}
 F(o(a)) &= \underbrace{D_{KL} [q(s, \pi) \parallel p(s, \pi | o(a))]}_{\text{Divergence}} - \underbrace{\ln p(o(a))}_{(\log) \text{ Evidence}} \\
 &= \underbrace{E_q [\ln q(s, \pi)]}_{\text{Entropy}} - \underbrace{E_q [\ln p(s, \pi, o(a))]}_{\text{Energy}} \\
 &= \underbrace{D_{KL} [q(s, \pi) \parallel p(s, \pi)]}_{\text{Complexity}} - \underbrace{E_q [\ln p(o(a) | s, \pi)]}_{\text{Accuracy}}
 \end{aligned}$$

Expected free energy

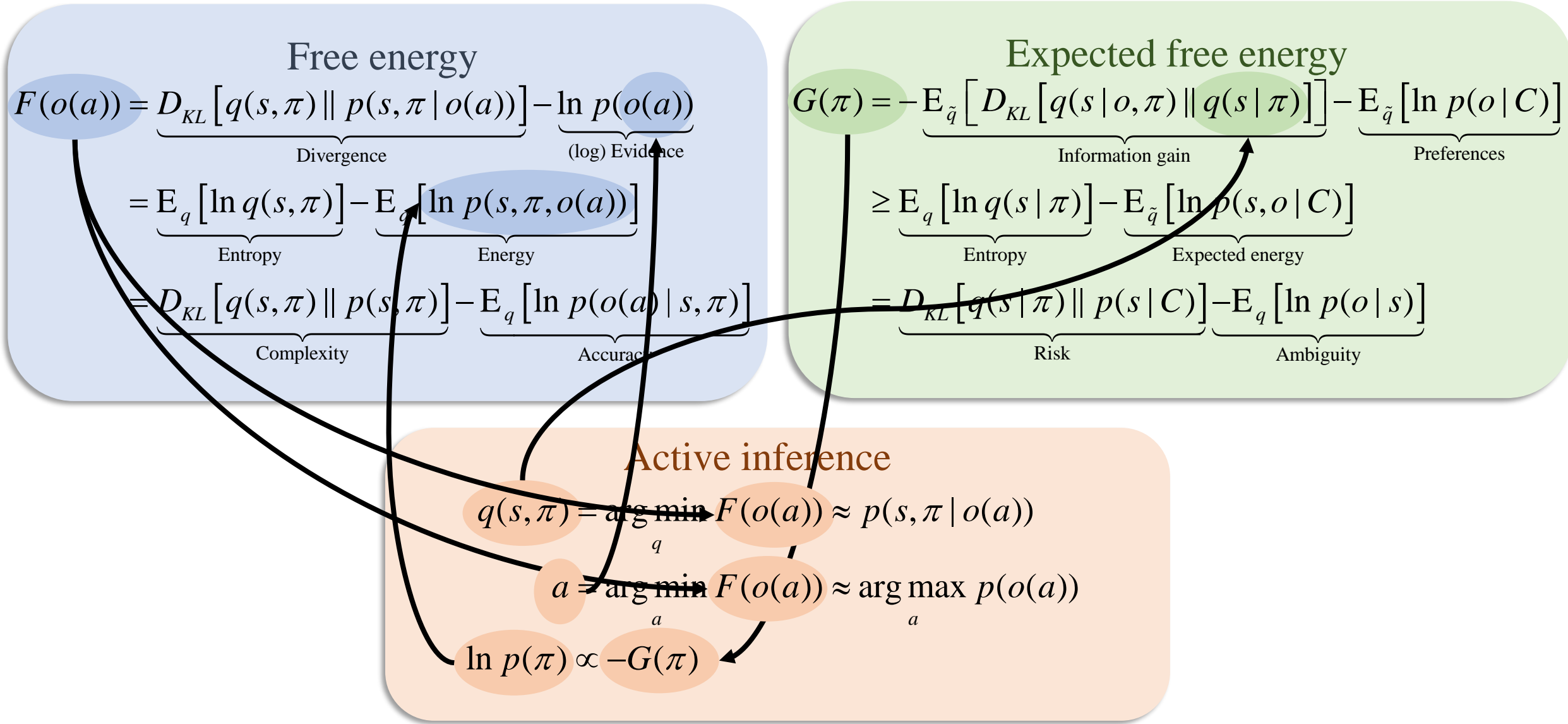
$$\begin{aligned}
 G(\pi) &= -E_{\tilde{q}} \left[\underbrace{D_{KL} [q(s | o, \pi) \parallel q(s | \pi)]}_{\text{Information gain}} \right] - \underbrace{E_{\tilde{q}} [\ln p(o | C)]}_{\text{Preferences}} \\
 &\geq \underbrace{E_q [\ln q(s | \pi)]}_{\text{Entropy}} - \underbrace{E_{\tilde{q}} [\ln p(s, o | C)]}_{\text{Expected energy}} \\
 &= \underbrace{D_{KL} [q(s | \pi) \parallel p(s | C)]}_{\text{Risk}} - \underbrace{E_q [\ln p(o | s)]}_{\text{Ambiguity}}
 \end{aligned}$$

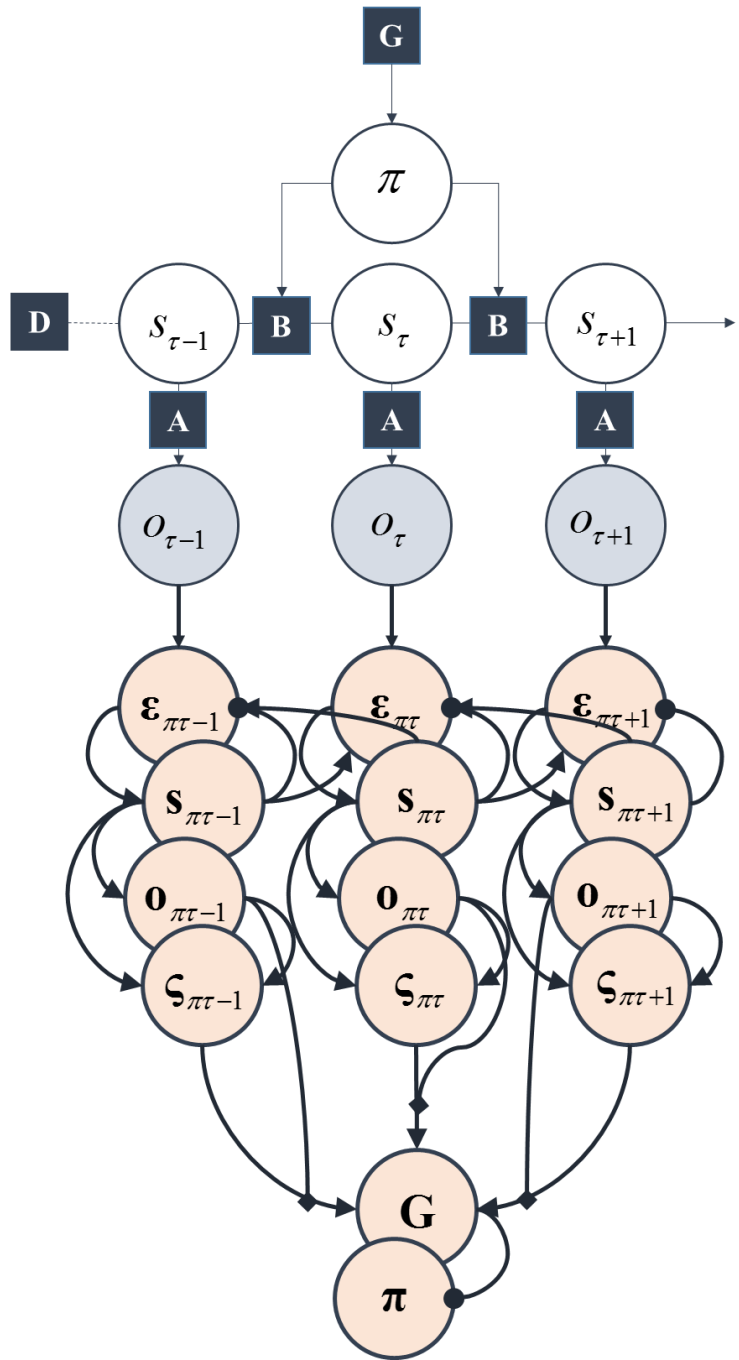
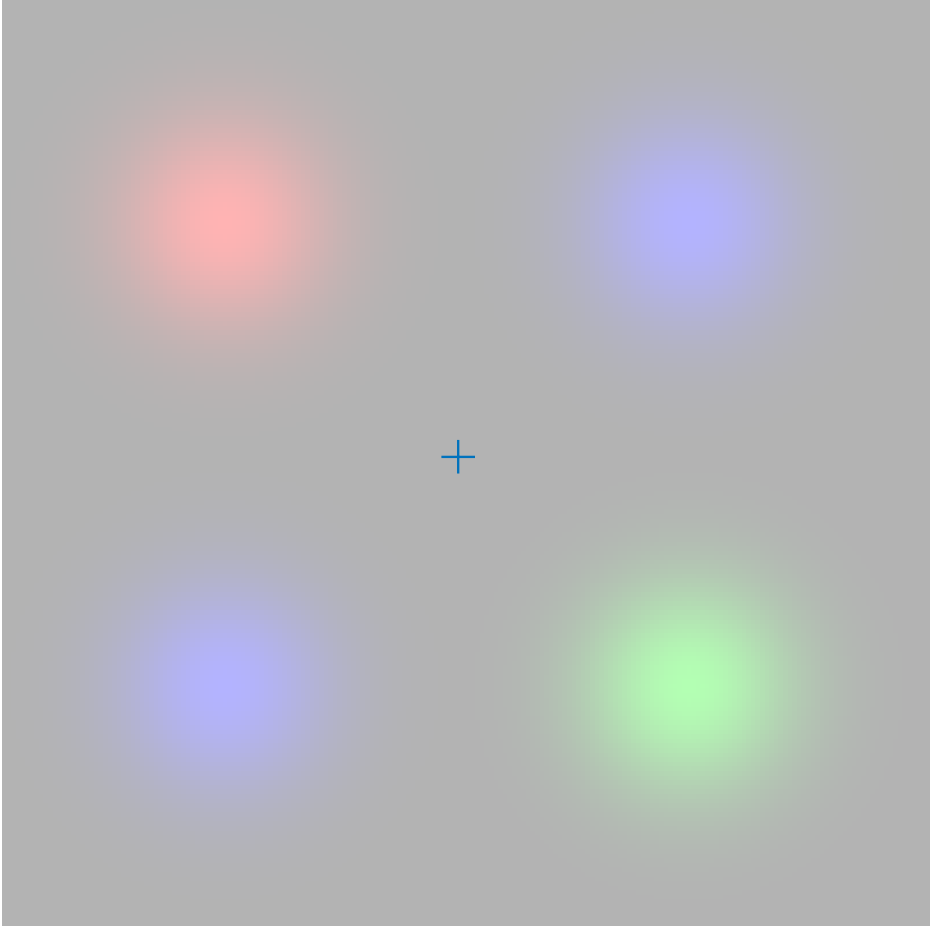
Active inference

$$q(s, \pi) = \arg \min_q F(o(a)) \approx p(s, \pi | o(a))$$

$$a = \arg \min_a F(o(a)) \approx \arg \max_a p(o(a))$$

$$\ln p(\pi) \propto -G(\pi)$$





Generative model

$$P(\tilde{o}, \tilde{s}, \pi) = P(s_1)P(\pi) \prod_{\tau} P(s_{\tau+1} | s_{\tau}, \pi) P(o_{\tau} | s_{\tau})$$

$$P(o_{\tau} | s_{\tau}) = \text{Cat}(\mathbf{A})$$

$$P(s_{\tau} | s_{\tau-1}, \pi) = \text{Cat}(\mathbf{B}_{\pi_{\tau}})$$

$$P(o_{\tau}) = \text{Cat}(\mathbf{C})$$

$$P(s_1) = \text{Cat}(\mathbf{D})$$

$$P(\pi) = \sigma(-\mathbf{G})$$

Bayesian message passing

$$\mathbf{s}_{\tau} = \boldsymbol{\pi} \cdot \mathbf{s}_{\pi_{\tau}}$$

$$\mathbf{s}_{\pi_{\tau}} = \sigma(\mathbf{v}_{\pi_{\tau}}); \dot{\mathbf{v}}_{\pi_{\tau}} = \boldsymbol{\epsilon}_{\pi_{\tau}}$$

$$\boldsymbol{\epsilon}_{\pi_{\tau}} = \ln \mathbf{A} \cdot \mathbf{o}_{\tau} + \frac{1}{2} \ln(\mathbf{B}_{\pi_{\tau}} \mathbf{s}_{\pi_{\tau}}) + \frac{1}{2} \ln(\mathbf{B}_{\pi_{\tau+1}}^{\dagger} \mathbf{s}_{\pi_{\tau+1}}) - \ln \mathbf{s}_{\pi_{\tau}}$$

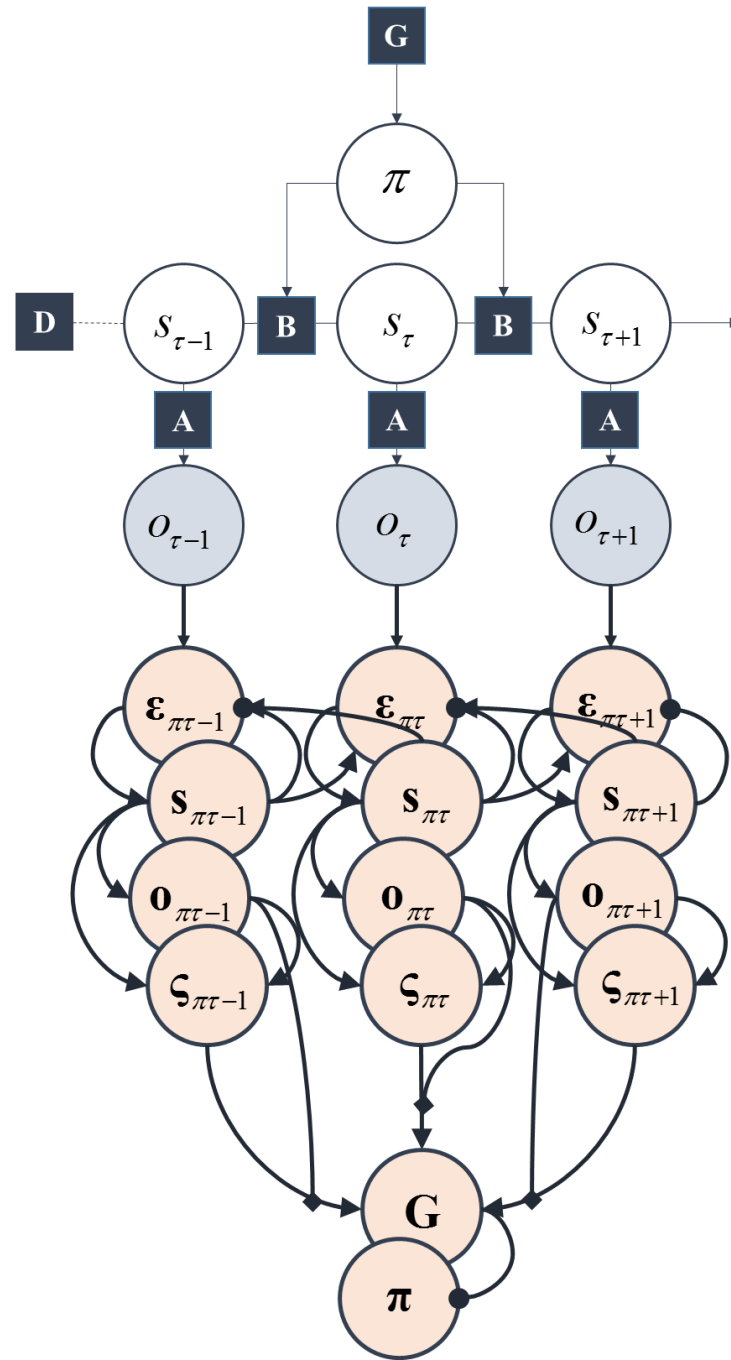
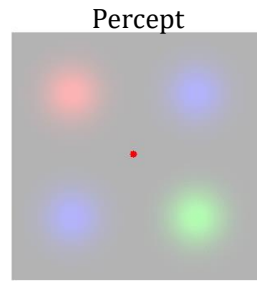
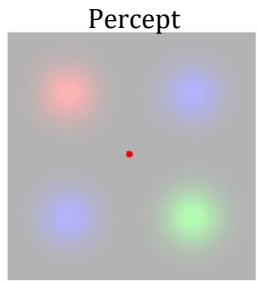
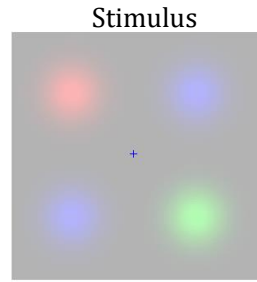
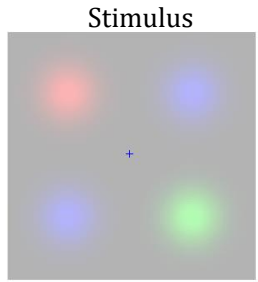
$$\mathbf{o}_{\pi_{\tau}} = \mathbf{A} \mathbf{s}_{\pi_{\tau}}$$

$$\boldsymbol{\zeta}_{\pi_{\tau}} = \ln \mathbf{o}_{\pi_{\tau}} - \ln \mathbf{C} + \mathbf{H} \cdot \mathbf{s}_{\pi_{\tau}}$$

$$\mathbf{H} = -\text{diag}(\mathbf{A} \cdot \ln \mathbf{A})$$

$$\mathbf{G}_{\pi} = \mathbf{o}_{\pi_{\tau}} \cdot \boldsymbol{\zeta}_{\pi_{\tau}}$$

$$\boldsymbol{\pi} = \sigma(-\mathbf{G})$$



Generative model

$$P(\tilde{o}, \tilde{s}, \pi) = P(s_1)P(\pi) \prod_{\tau} P(s_{\tau+1} | s_{\tau}, \pi) P(o_{\tau} | s_{\tau})$$

$$P(o_{\tau} | s_{\tau}) = \text{Cat}(\mathbf{A})$$

$$P(s_{\tau} | s_{\tau-1}, \pi) = \text{Cat}(\mathbf{B}_{\pi\tau})$$

$$P(o_{\tau}) = \text{Cat}(\mathbf{C})$$

$$P(s_1) = \text{Cat}(\mathbf{D})$$

$$P(\pi) = \sigma(-\mathbf{G})$$

Bayesian message passing

$$\mathbf{s}_{\tau} = \boldsymbol{\pi} \cdot \mathbf{s}_{\pi\tau}$$

$$\mathbf{s}_{\pi\tau} = \sigma(\mathbf{v}_{\pi\tau}); \dot{\mathbf{v}}_{\pi\tau} = \boldsymbol{\varepsilon}_{\pi\tau}$$

$$\boldsymbol{\varepsilon}_{\pi\tau} = \ln \mathbf{A} \cdot \mathbf{o}_{\tau} + \frac{1}{2} \ln(\mathbf{B}_{\pi\tau} \cdot \mathbf{s}_{\pi\tau}) + \frac{1}{2} \ln(\mathbf{B}_{\pi\tau+1}^{\dagger} \cdot \mathbf{s}_{\pi\tau+1}) - \ln \mathbf{s}_{\pi\tau}$$

$$\mathbf{o}_{\pi\tau} = \mathbf{A} \mathbf{s}_{\pi\tau}$$

$$\boldsymbol{\zeta}_{\pi\tau} = \ln \mathbf{o}_{\pi\tau} - \ln \mathbf{C} + \mathbf{H} \cdot \mathbf{s}_{\pi\tau}$$

$$\mathbf{H} = -\text{diag}(\mathbf{A} \cdot \ln \mathbf{A})$$

$$\mathbf{G}_{\pi} = \mathbf{o}_{\pi\tau} \cdot \boldsymbol{\zeta}_{\pi\tau}$$

$$\boldsymbol{\pi} = \sigma(-\mathbf{G})$$



Active Inference

Self-evidencing

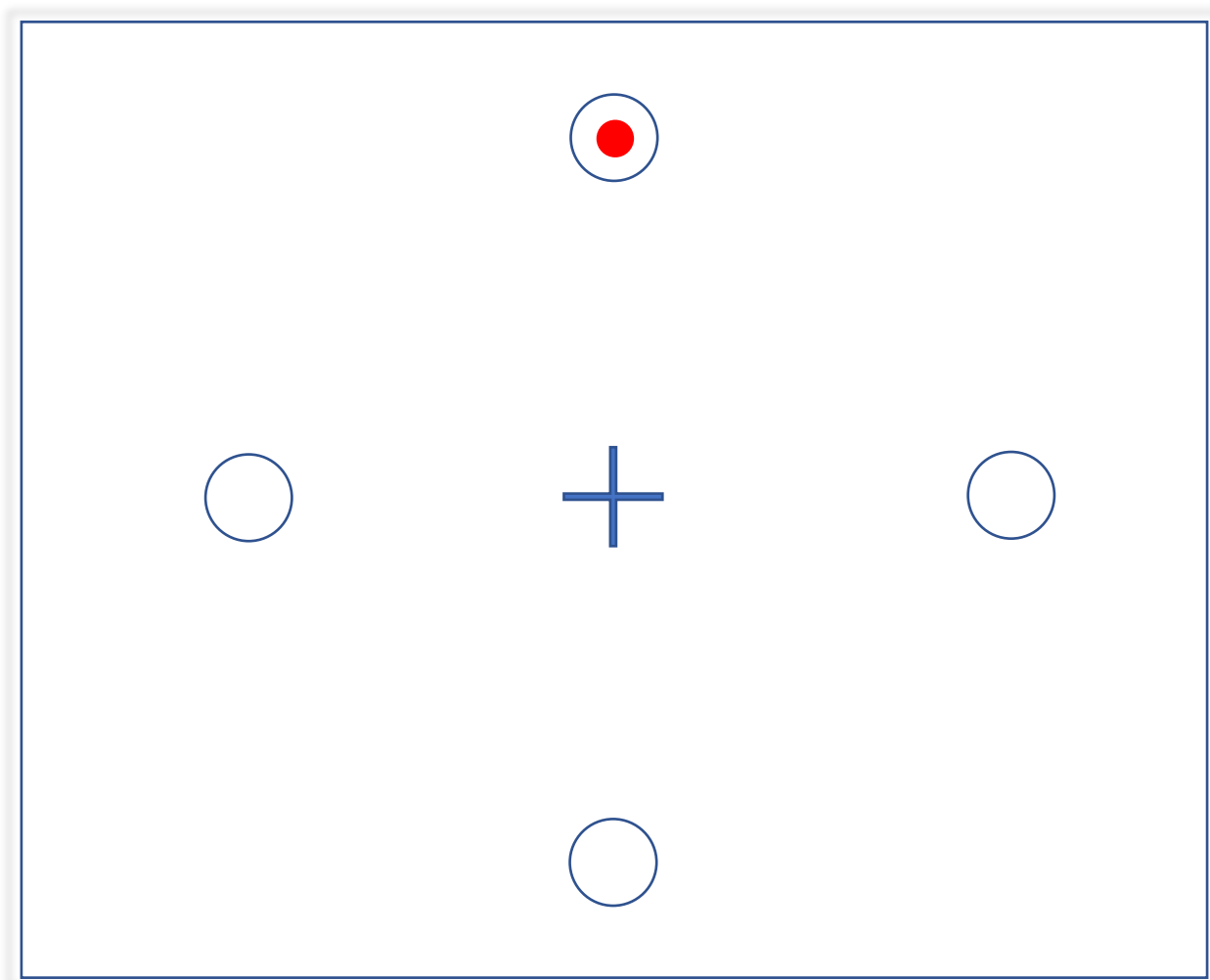
Message passing

Discrete time (planning)

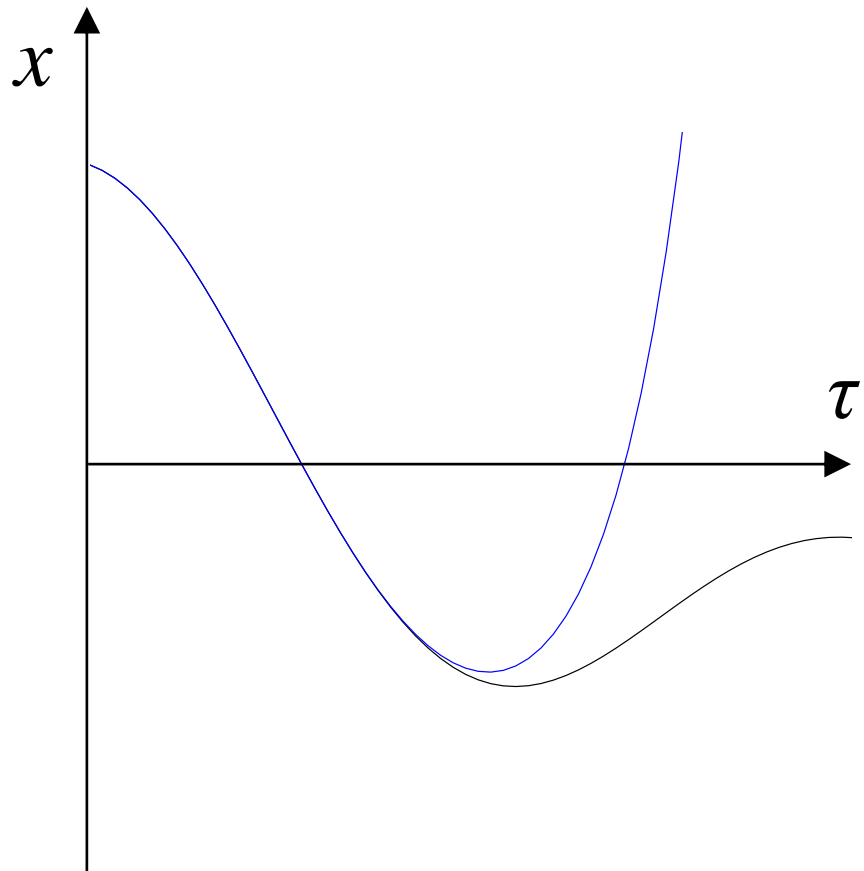
Continuous time (movement)

Hierarchical models

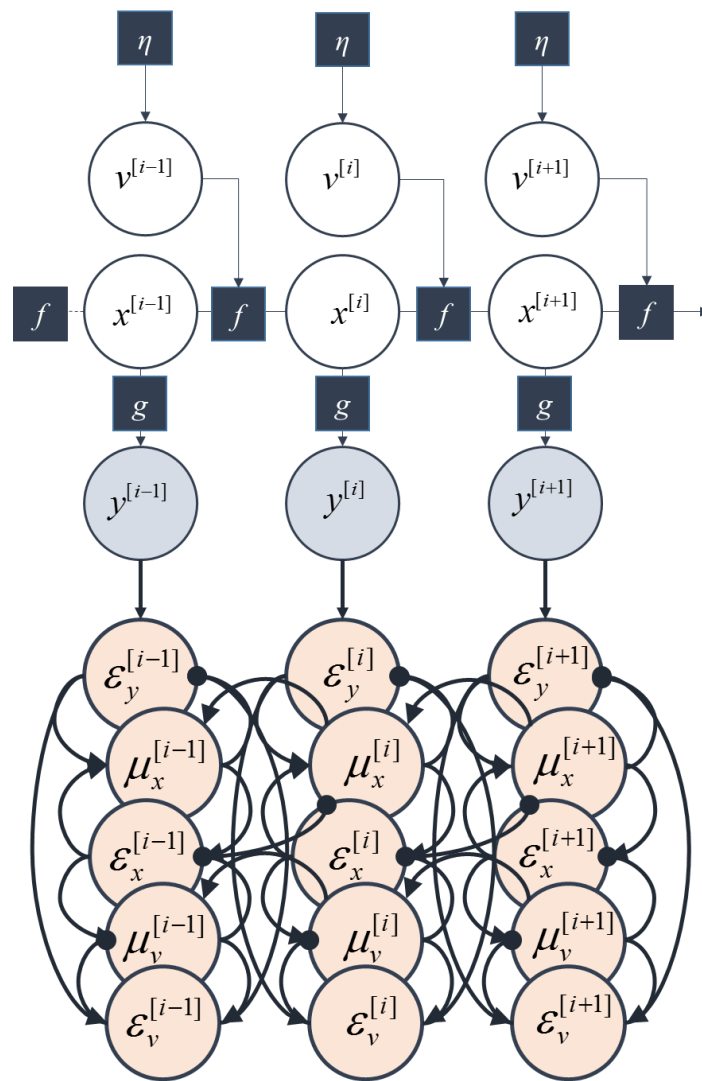
Summary



Continuous time models



$$x(\tau) \approx x_0 + \tau x'_0 + \frac{1}{2} \tau^2 x''_0 + \frac{1}{6} \tau^3 x'''_0 + \dots$$



Generative model

$$p(\tilde{y}, \tilde{x}, \tilde{v}) = \prod_i p(v^{[i]}) p(x^{[i+1]} | x^{[i]}, v^{[i]}) p(y^{[i]} | x^{[i]}, v^{[i]})$$

$$p(y^{[i]} | x^{[i]}, v^{[i]}) = \mathcal{N}(g^{[i]}(x^{[i]}, v^{[i]}), \Pi_y^{[i]})$$

$$p(x^{[i+1]} | x^{[i]}, v^{[i]}) = \mathcal{N}(f^{[i]}(x^{[i]}, v^{[i]}), \Pi_x^{[i]})$$

$$p(v^{[i]}) = \mathcal{N}(\eta^{[i]}, \Pi_v^{[i]})$$

Bayesian message passing

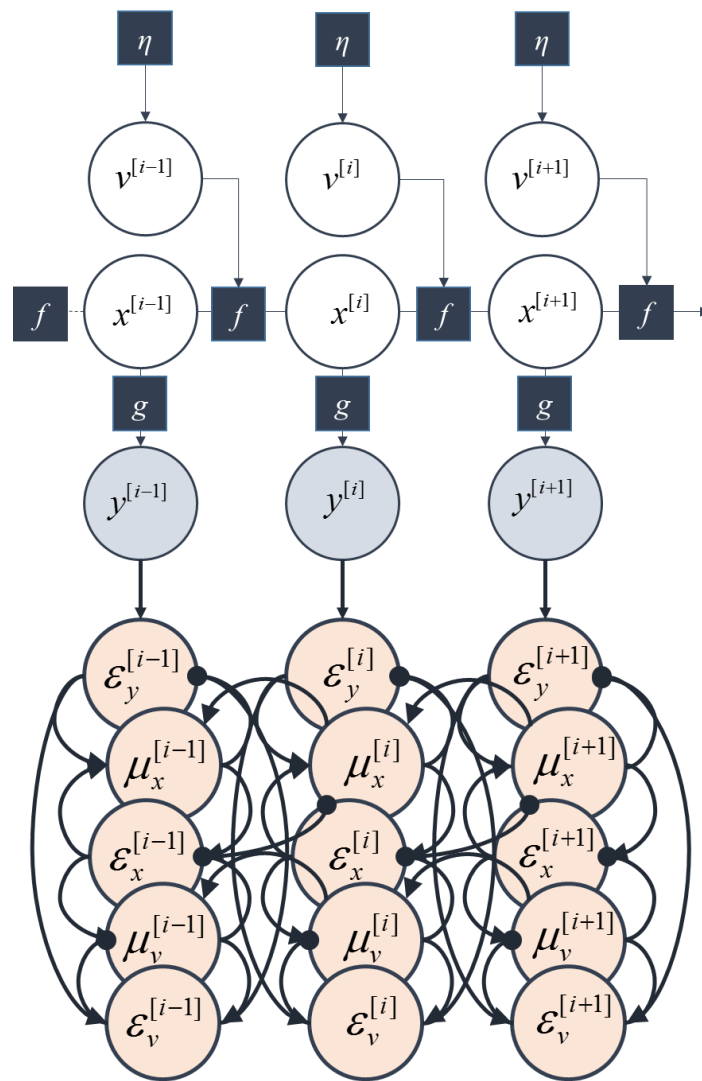
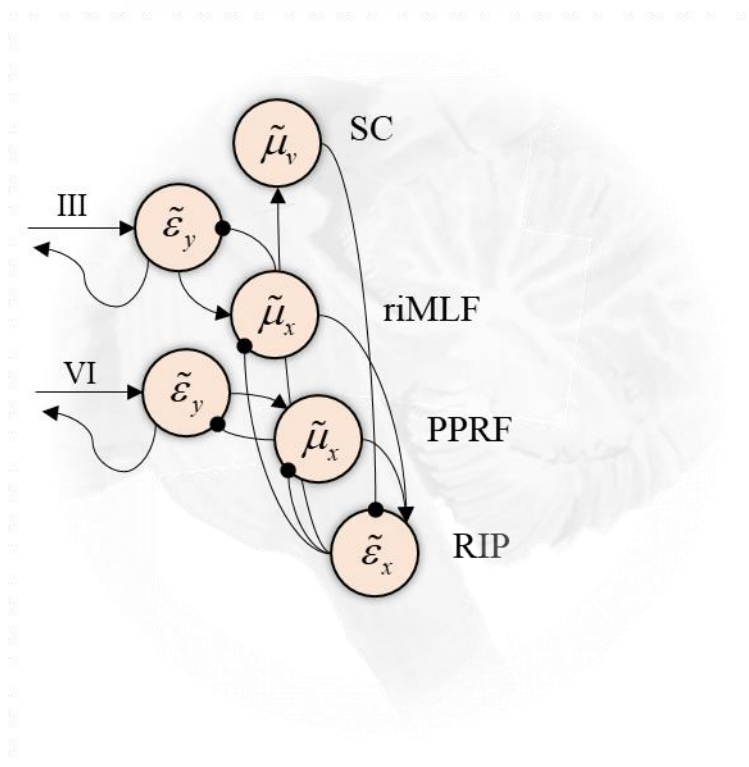
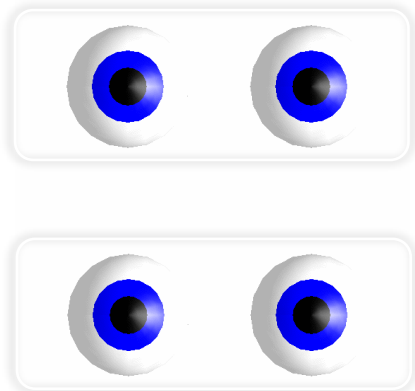
$$\varepsilon_y^{[i]} = y^{[i]} - g^{[i]}(\mu_x^{[i]}, \mu_v^{[i]})$$

$$\varepsilon_x^{[i]} = \mu_x^{[i+1]} - f^{[i]}(\mu_x^{[i]}, \mu_v^{[i]})$$

$$\varepsilon_v^{[i]} = \mu_v^{[i]} - \eta^{[i]}$$

$$\begin{aligned} \dot{\mu}_x^{[i]} &= \mu_x^{[i+1]} \\ &+ \partial_{\mu_x^{[i]}} g^{[i]} \cdot \Pi_y^{[i]} \varepsilon_y^{[i]} - \Pi_x^{[i-1]} \varepsilon_x^{[i-1]} + \partial_{\mu_x^{[i]}} f^{[i]} \cdot \Pi_x^{[i]} \varepsilon_x^{[i]} \end{aligned}$$

$$\begin{aligned} \dot{\mu}_v^{[i]} &= \mu_v^{[i+1]} \\ &+ \partial_{\mu_v^{[i]}} g^{[i]} \cdot \Pi_y^{[i]} \varepsilon_y^{[i]} + \partial_{\mu_v^{[i]}} f^{[i]} \cdot \Pi_x^{[i]} \varepsilon_x^{[i]} - \Pi_v^{[i]} \varepsilon_v^{[i]} \end{aligned}$$



Generative model

$$p(\tilde{y}, \tilde{x}, \tilde{v}) = \prod_i p(v^{[i]})p(x^{[i+1]} | x^{[i]}, v^{[i]})p(y^{[i]} | x^{[i]}, v^{[i]})$$

$$p(y^{[i]} | x^{[i]}, v^{[i]}) = \mathcal{N}(g^{[i]}(x^{[i]}, v^{[i]}), \Pi_y^{[i]})$$

$$p(x^{[i+1]} | x^{[i]}, v^{[i]}) = \mathcal{N}(f^{[i]}(x^{[i]}, v^{[i]}), \Pi_x^{[i]})$$

$$p(v^{[i]}) = \mathcal{N}(\eta^{[i]}, \Pi_v^{[i]})$$

Bayesian message passing

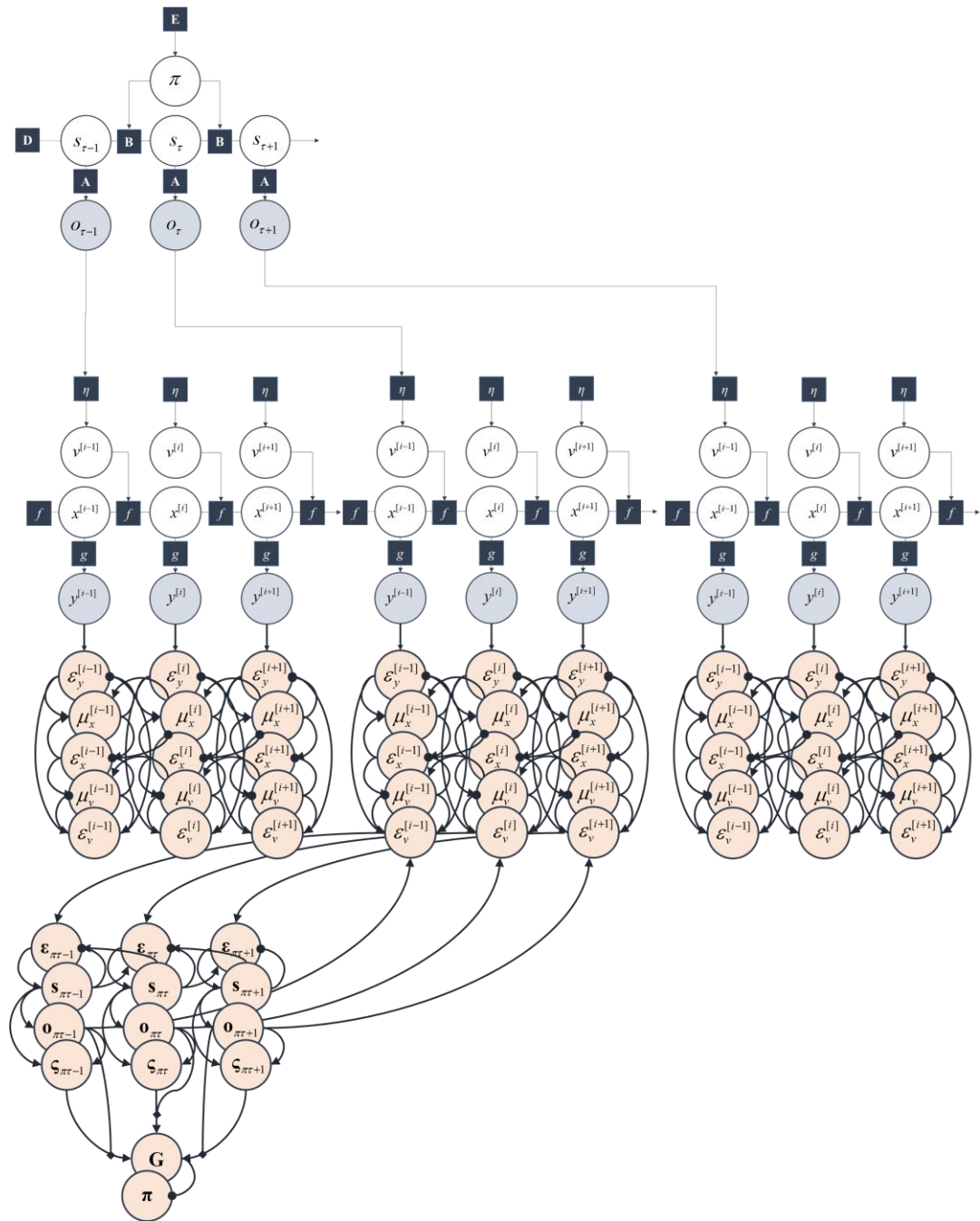
$$\epsilon_y^{[i]} = y^{[i]} - g^{[i]}(\mu_x^{[i]}, \mu_v^{[i]})$$

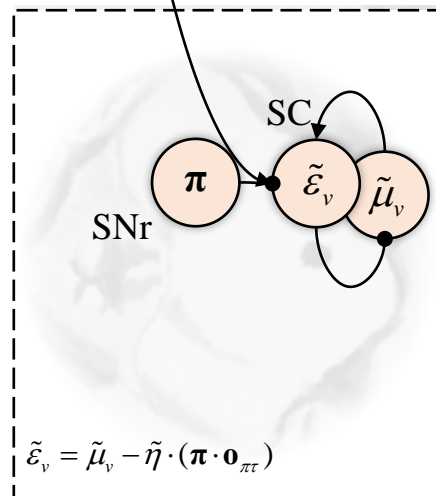
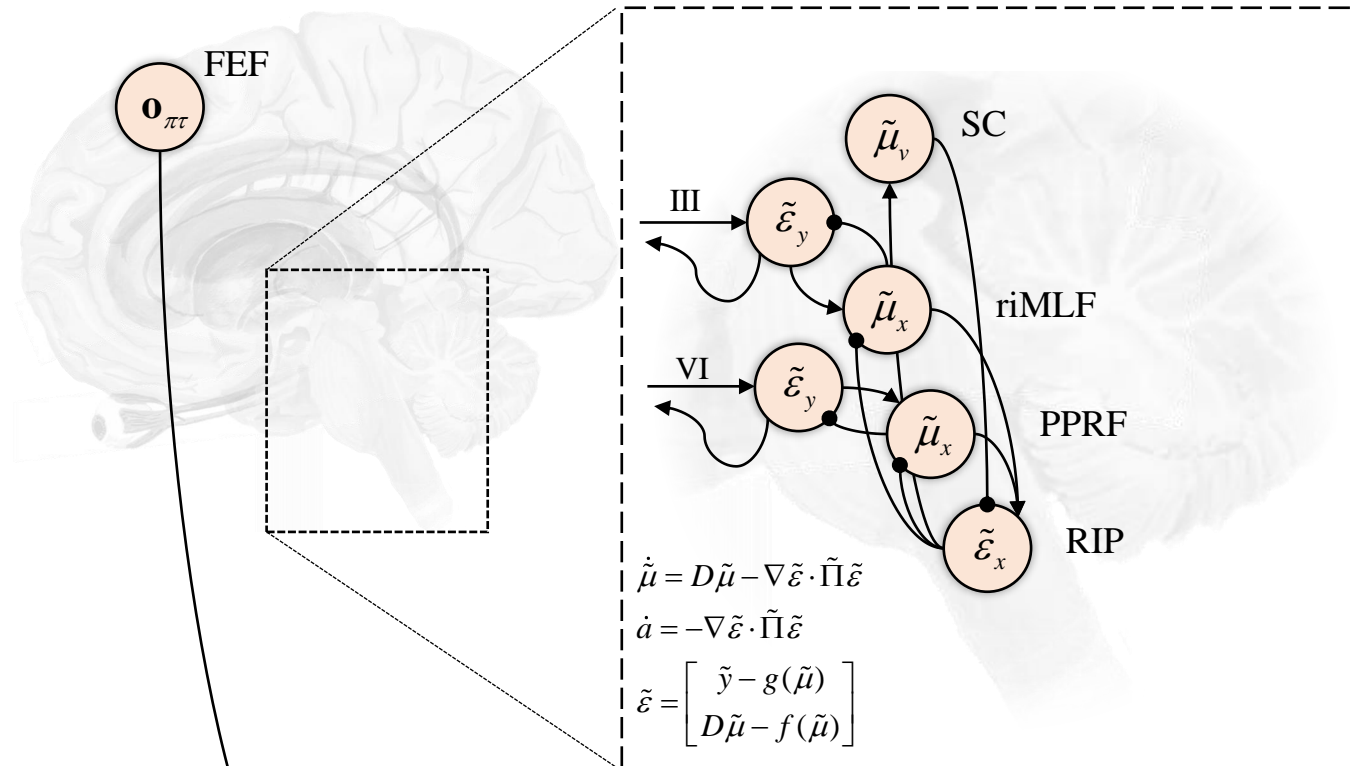
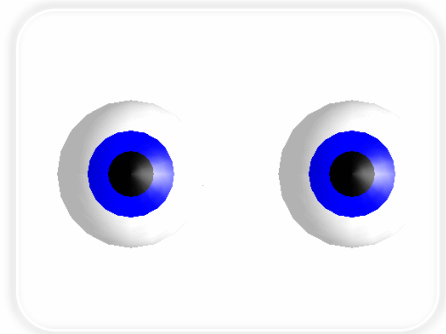
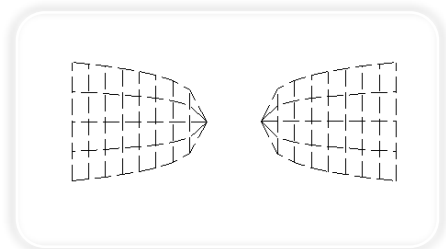
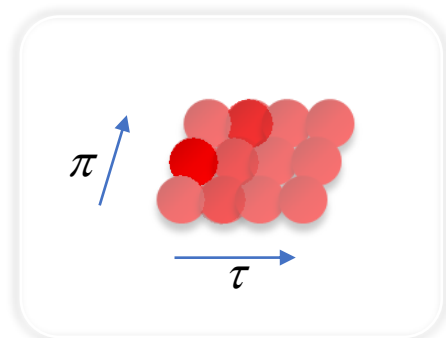
$$\epsilon_x^{[i]} = \mu_x^{[i+1]} - f^{[i]}(\mu_x^{[i]}, \mu_v^{[i]})$$

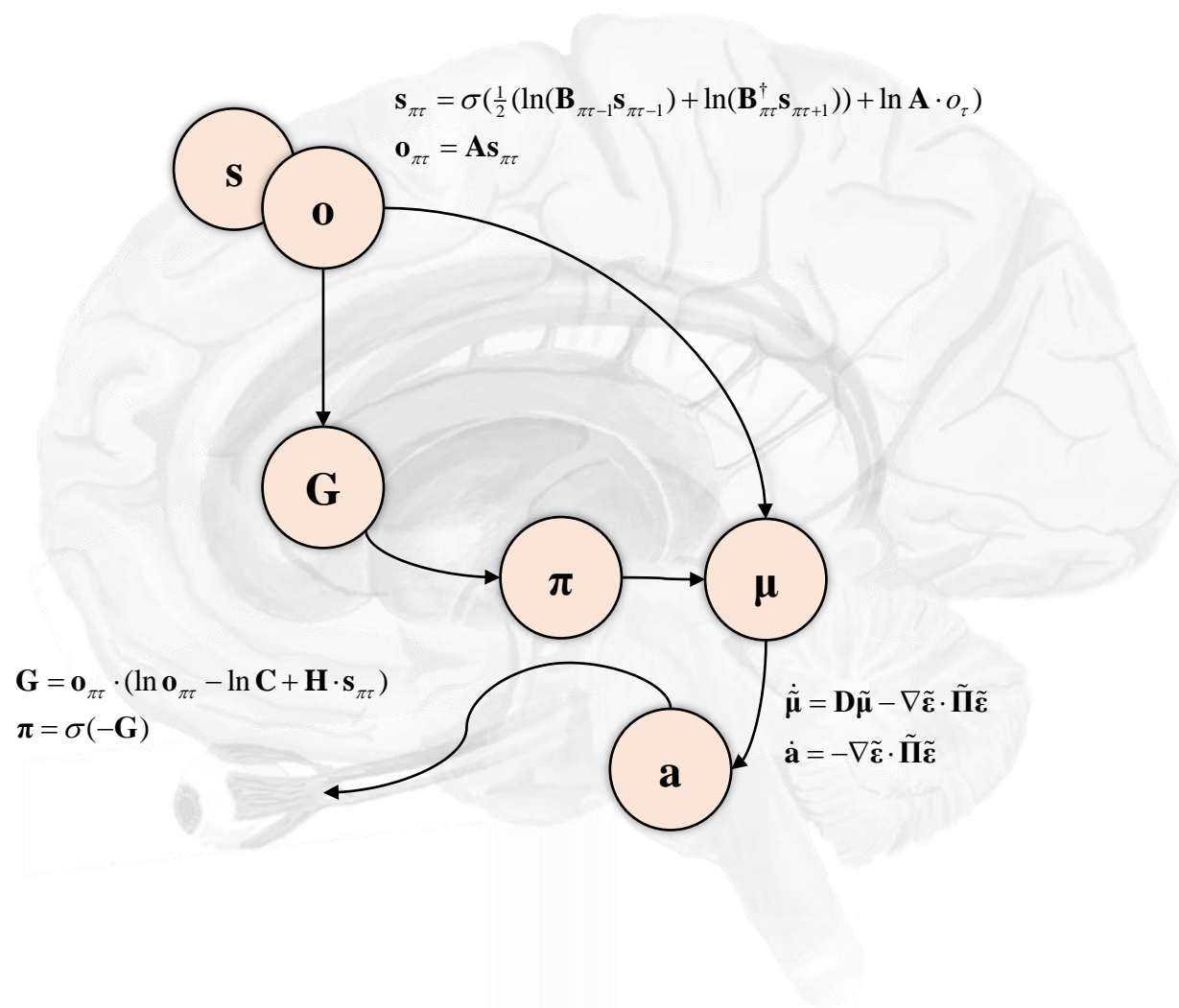
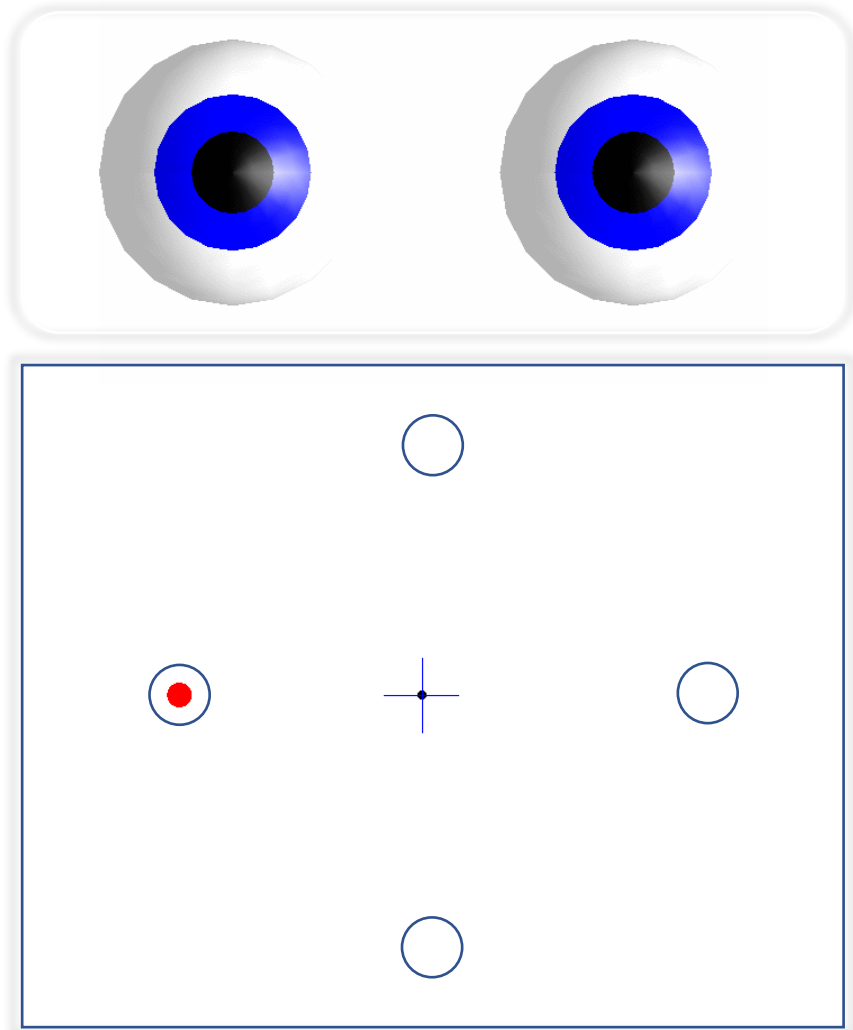
$$\epsilon_v^{[i]} = \mu_v^{[i]} - \eta^{[i]}$$

$$\dot{\mu}_x^{[i]} = \mu_x^{[i+1]} + \partial_{\mu_x^{[i]}} g^{[i]} \cdot \Pi_y^{[i]} \epsilon_y^{[i]} - \Pi_x^{[i-1]} \epsilon_x^{[i-1]} + \partial_{\mu_x^{[i]}} f^{[i]} \cdot \Pi_x^{[i]} \epsilon_x^{[i]}$$

$$\dot{\mu}_v^{[i]} = \mu_v^{[i+1]} + \partial_{\mu_v^{[i]}} g^{[i]} \cdot \Pi_y^{[i]} \epsilon_y^{[i]} + \partial_{\mu_v^{[i]}} f^{[i]} \cdot \Pi_x^{[i]} \epsilon_x^{[i]} - \Pi_v^{[i]} \epsilon_v^{[i]}$$









Active Inference

Self-evidencing

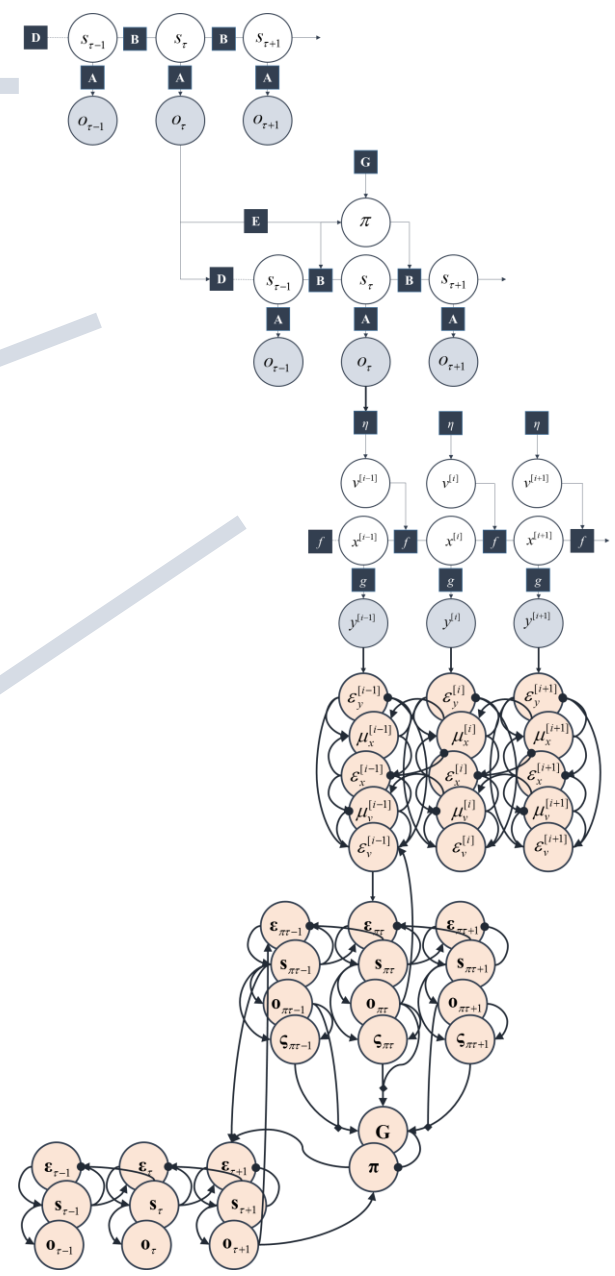
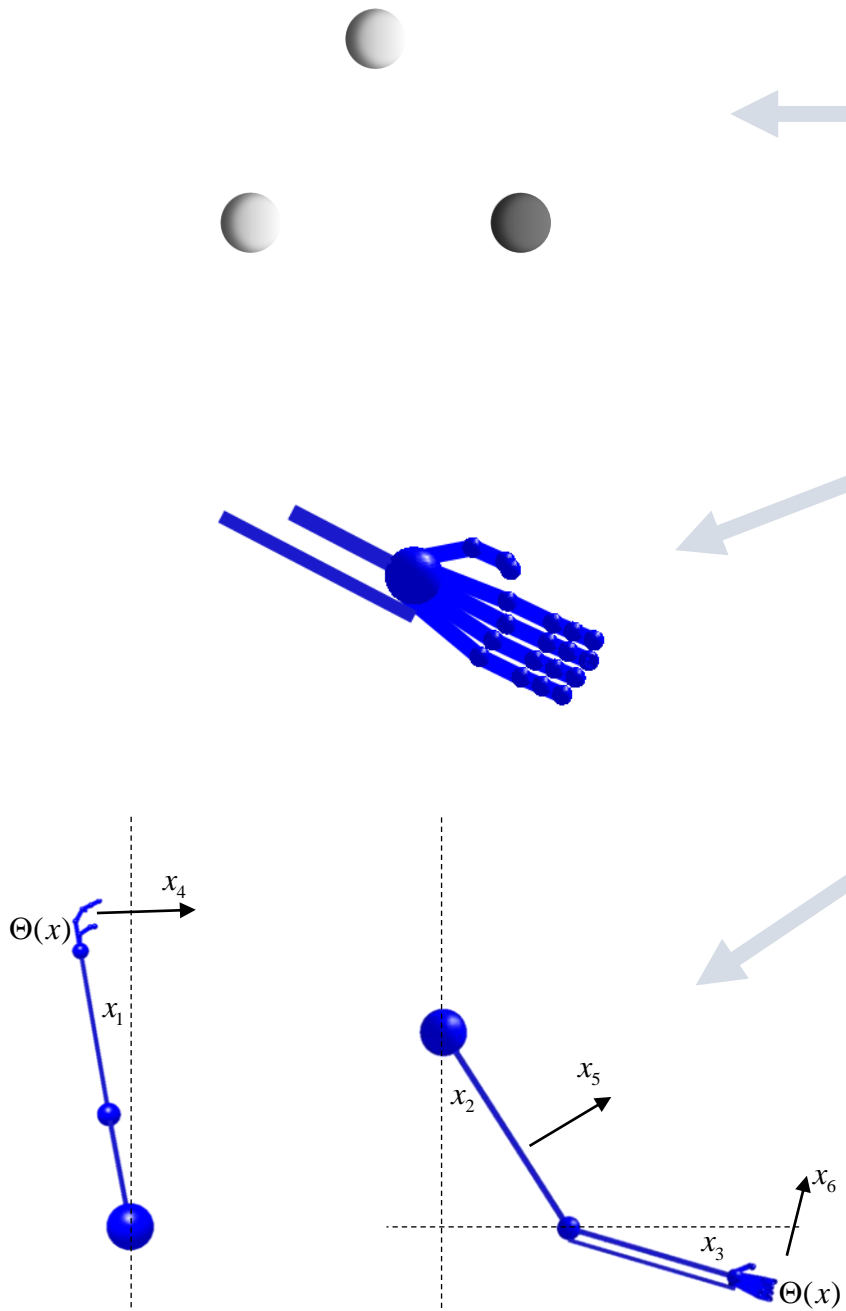
Message passing

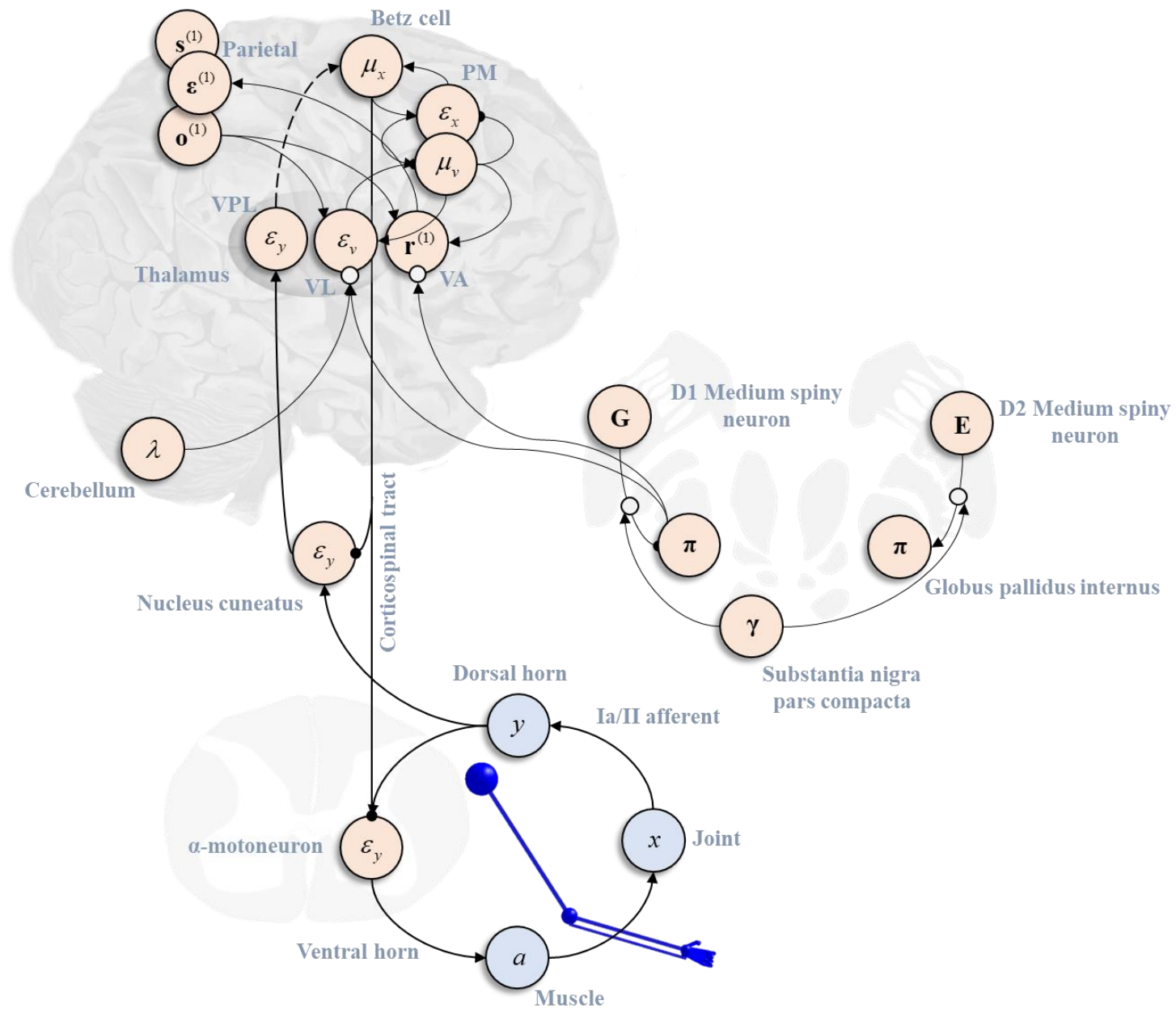
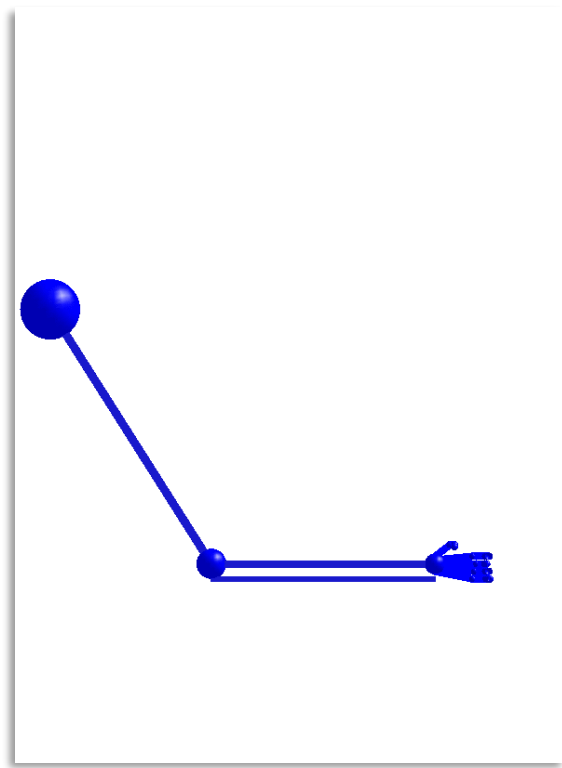
Discrete time (planning)

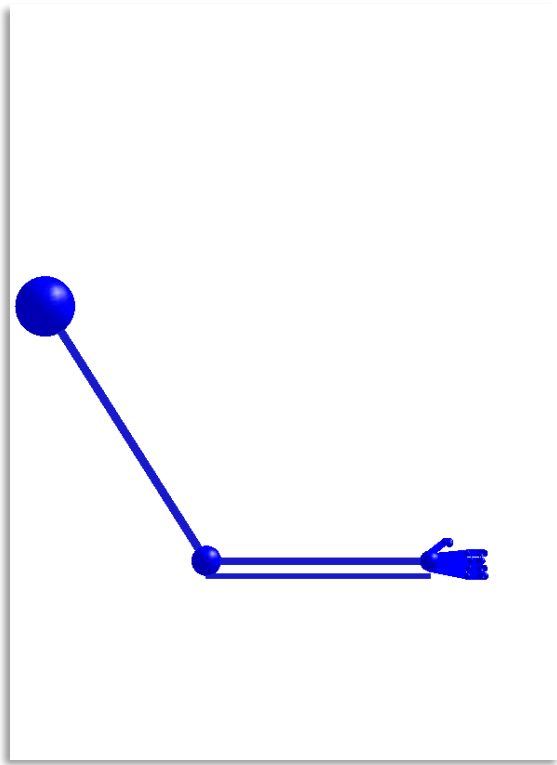
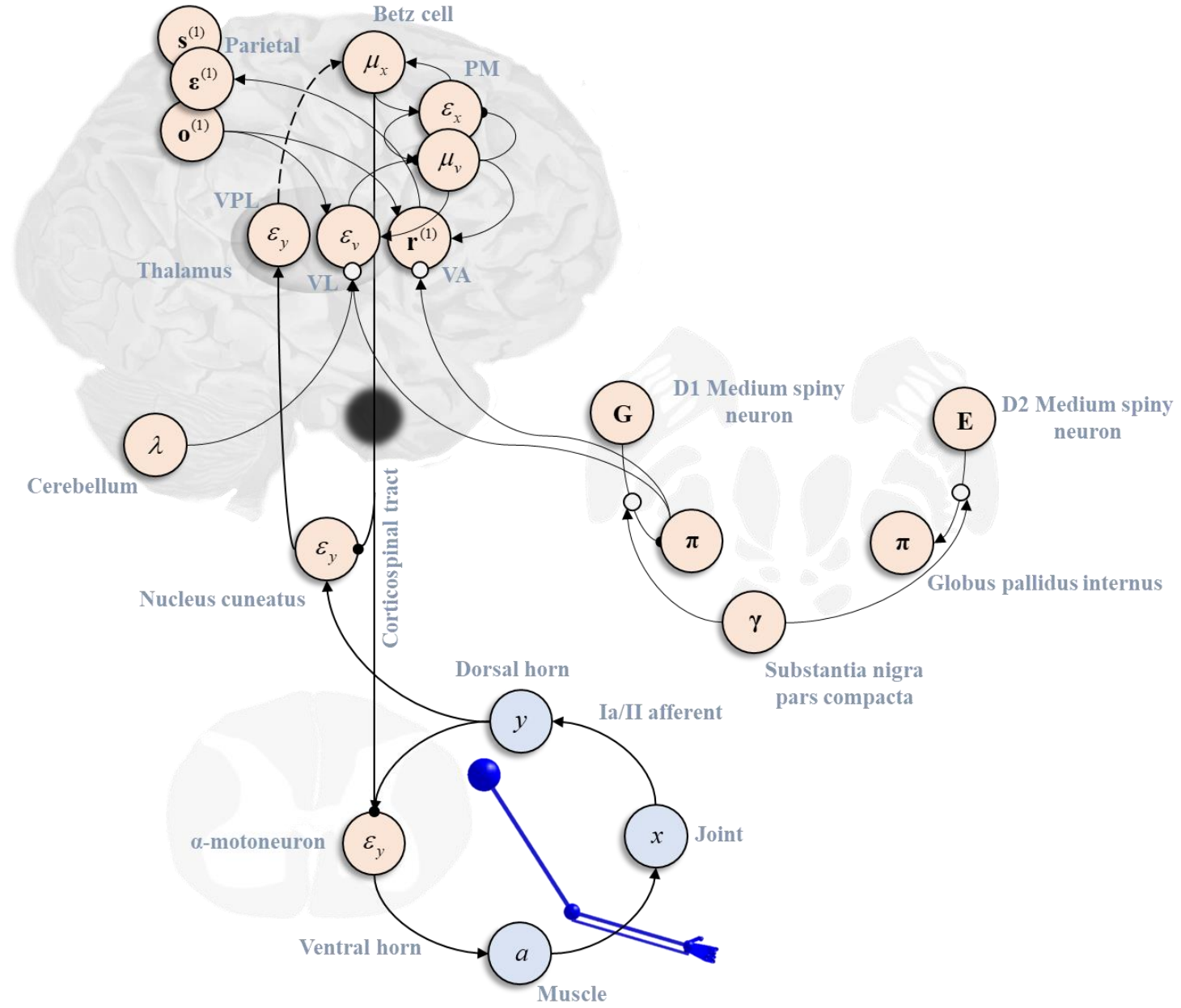
Continuous time (movement)

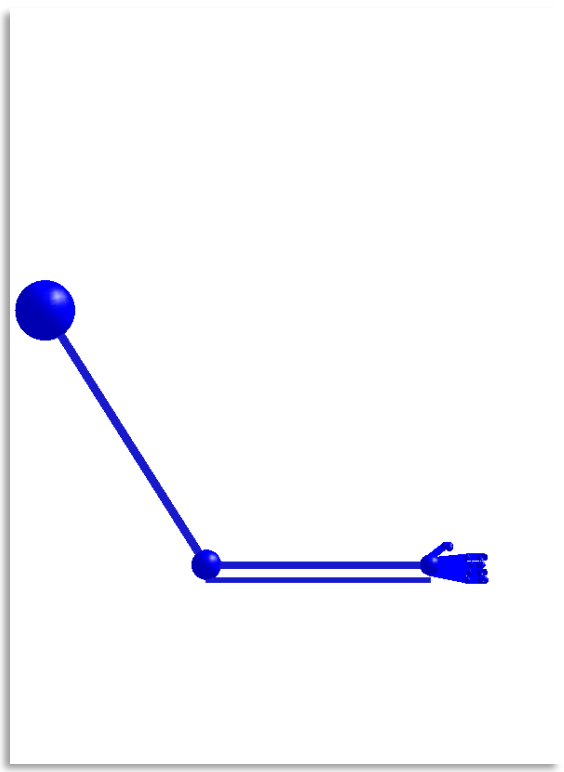
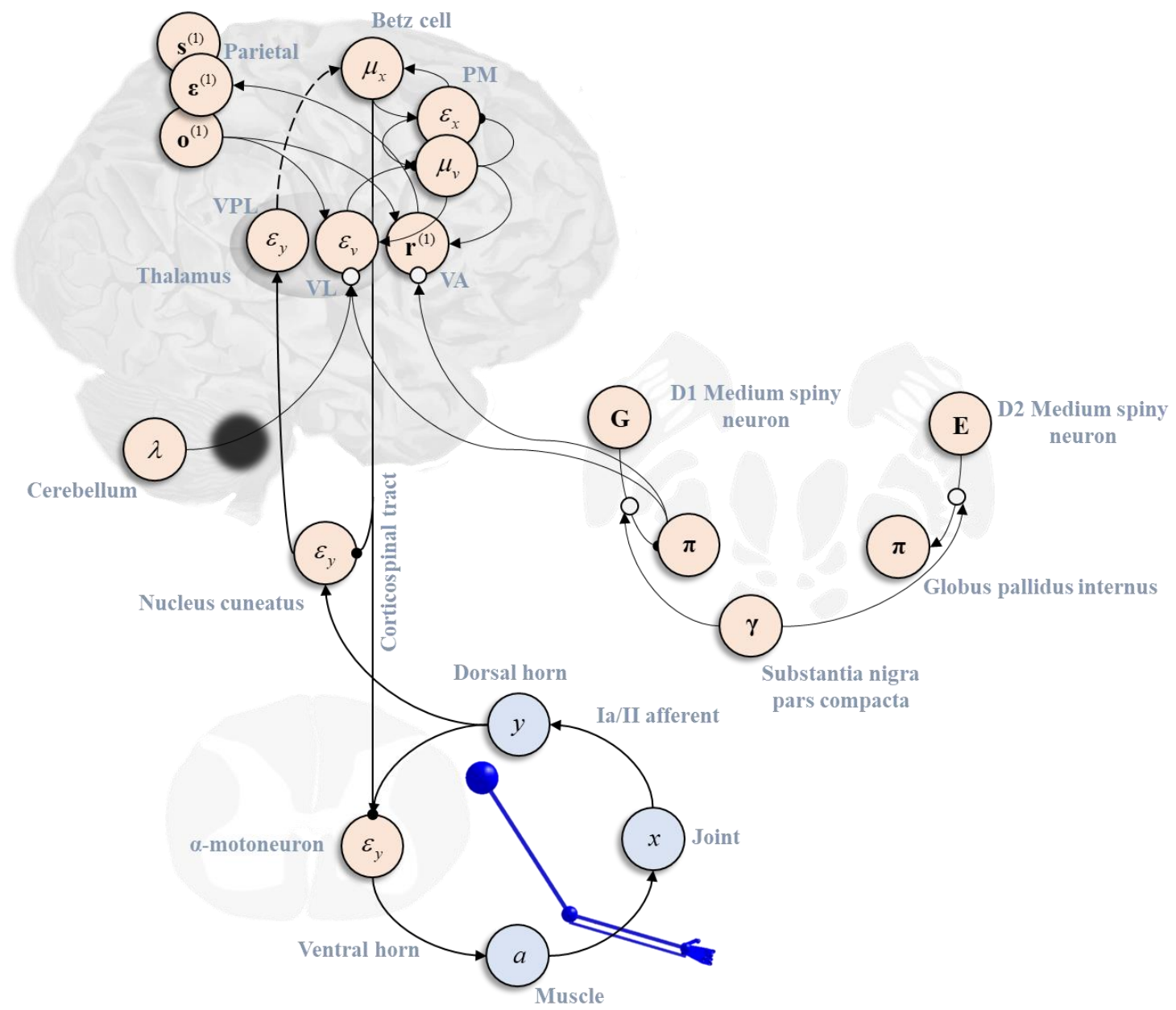
Hierarchical models

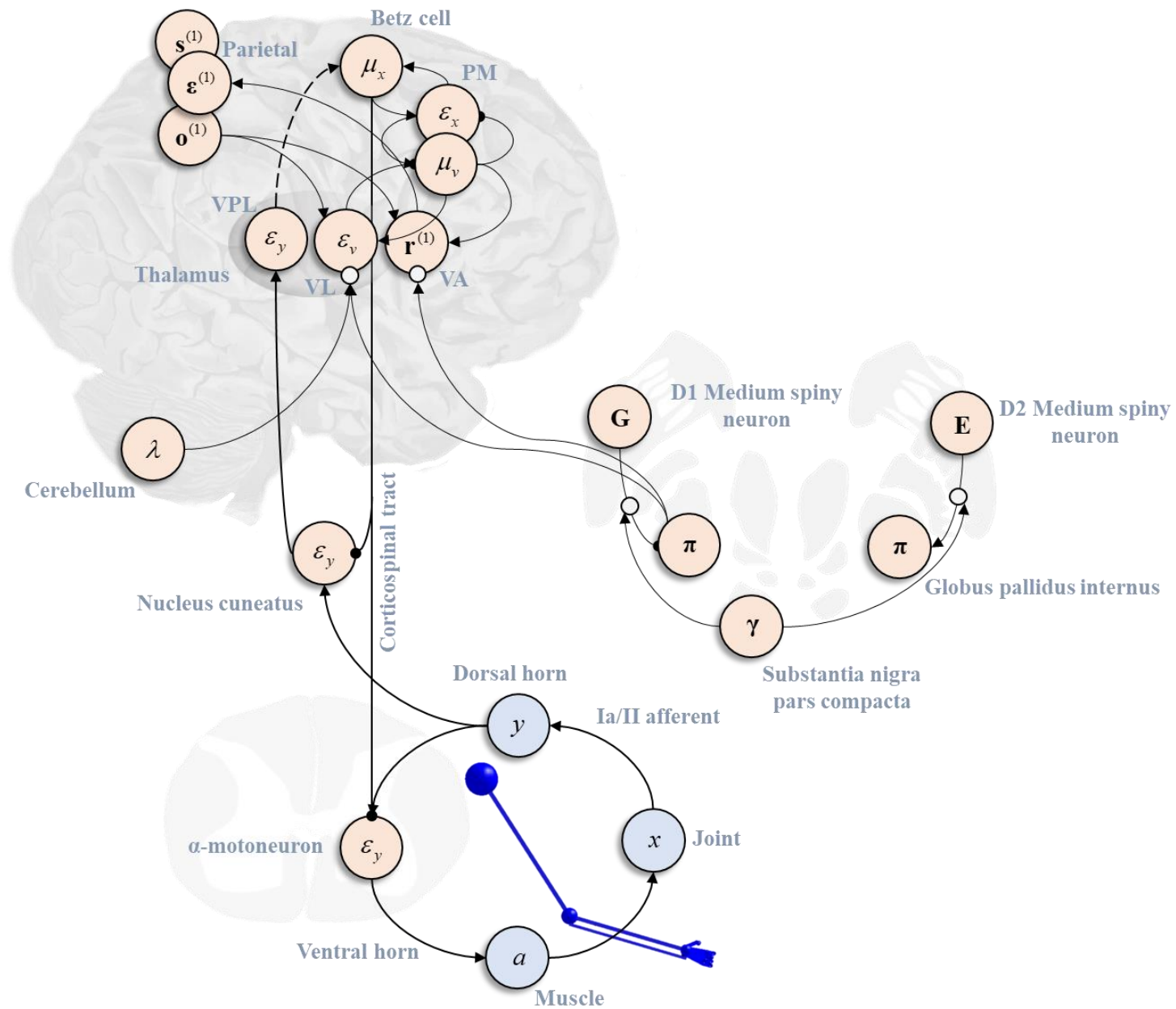
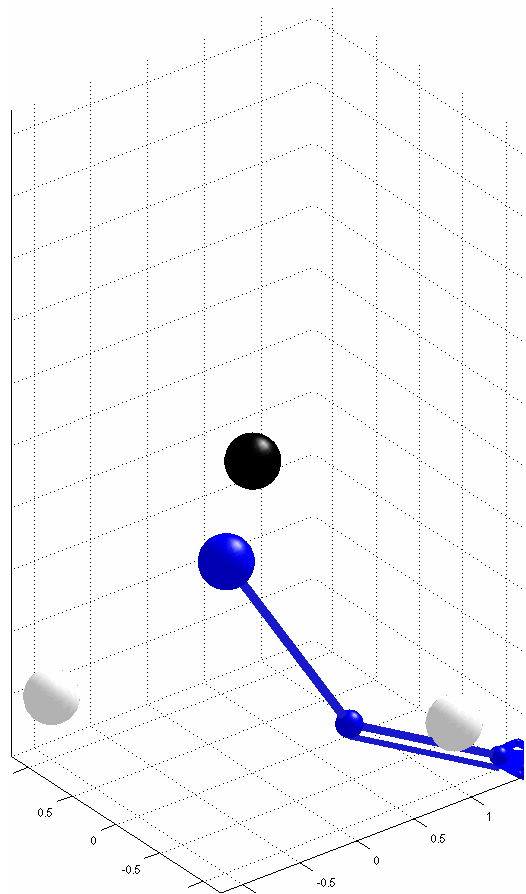
Summary

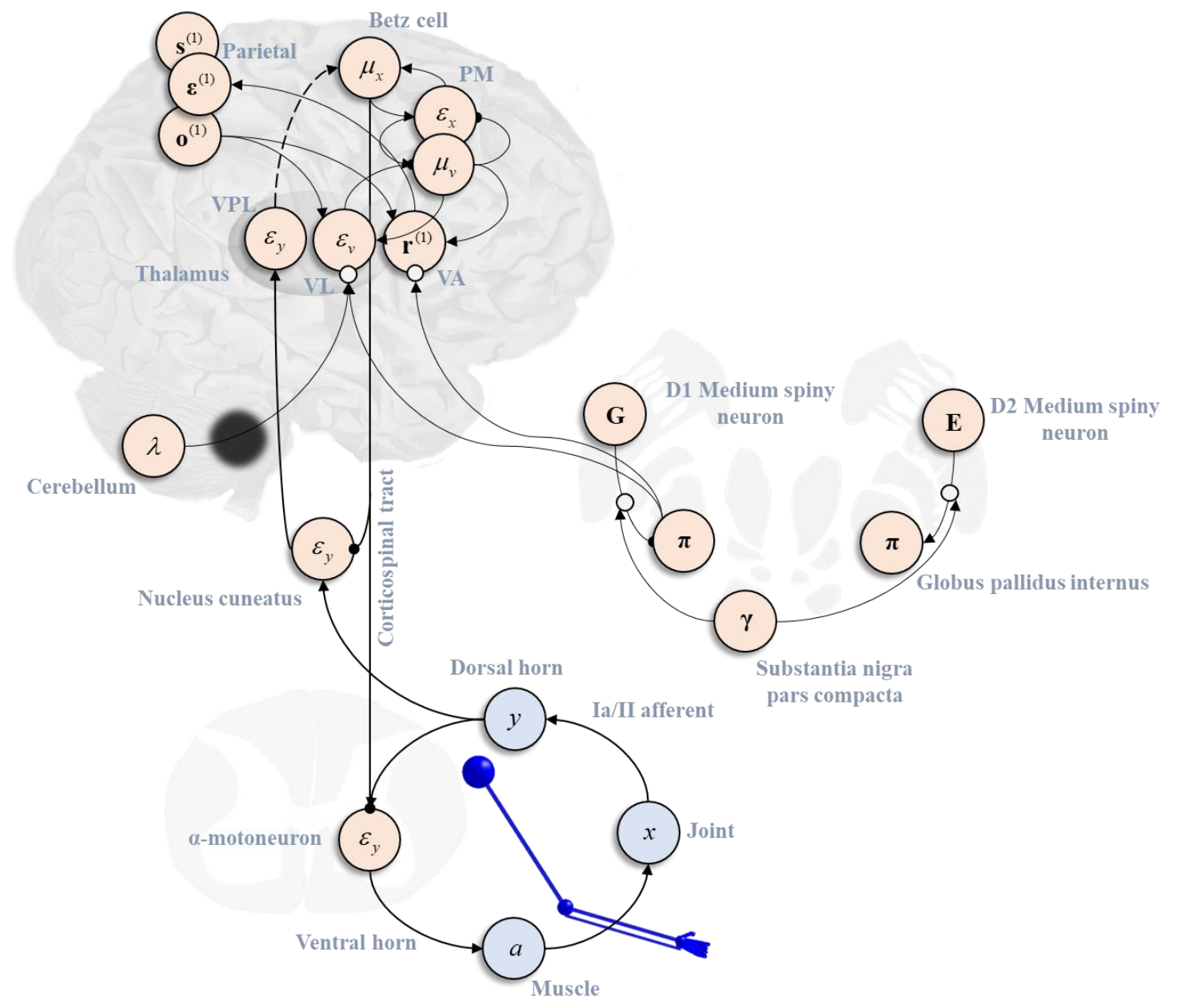
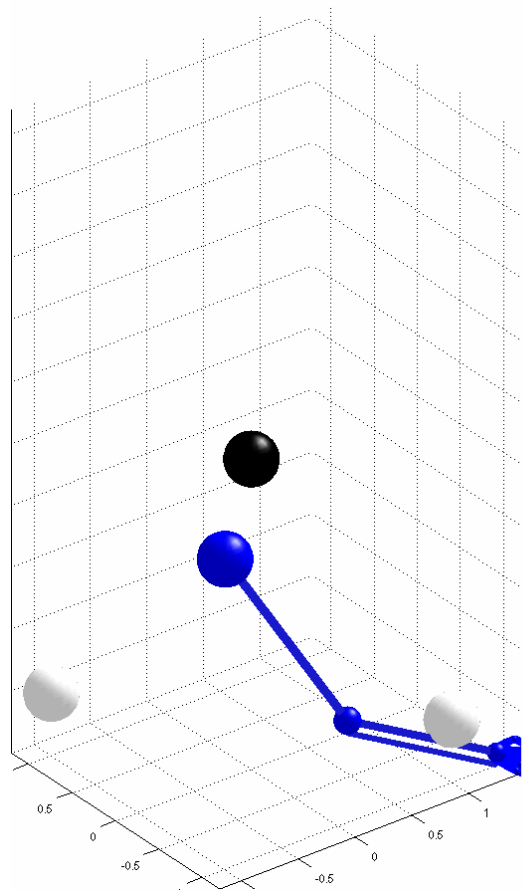


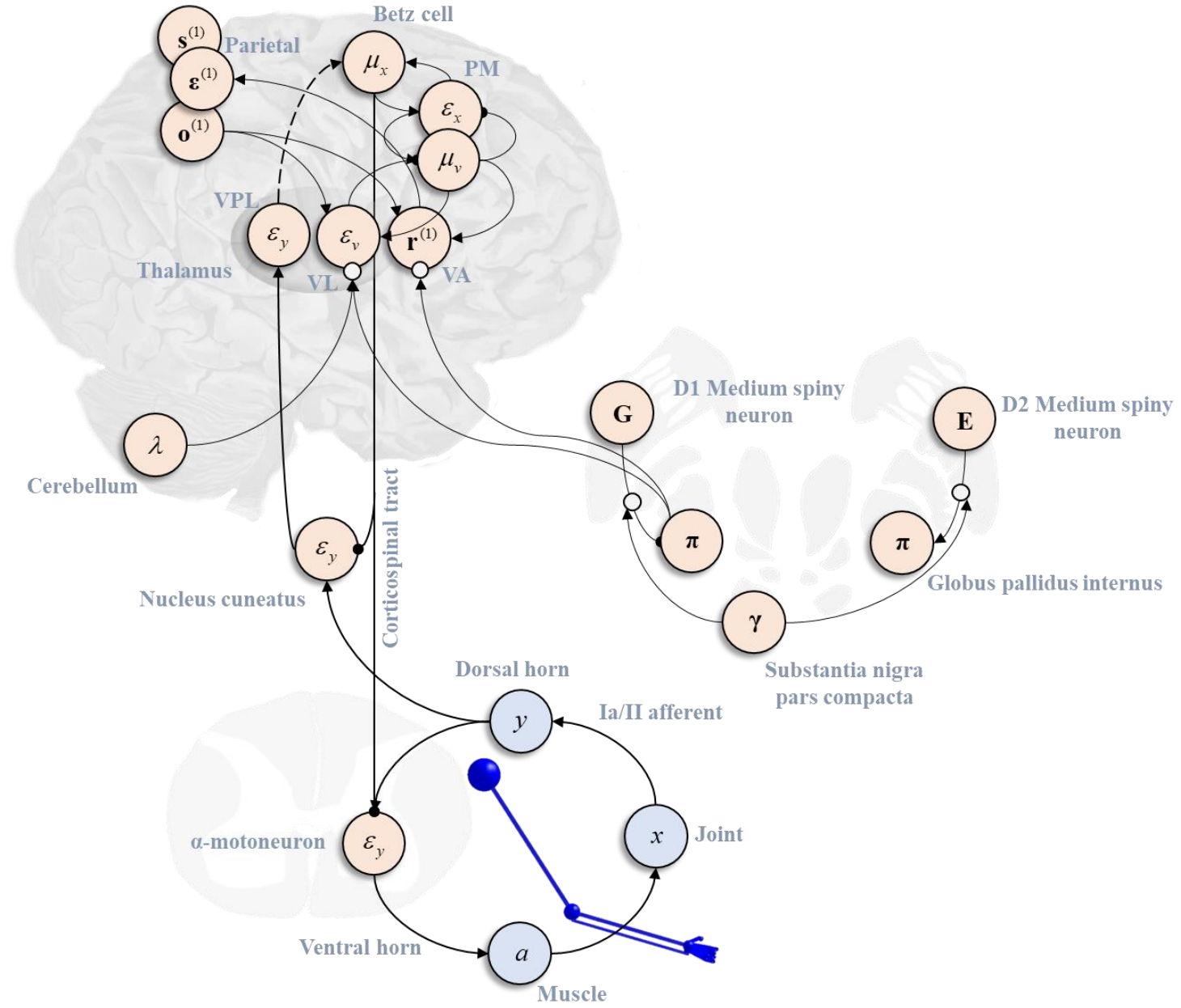


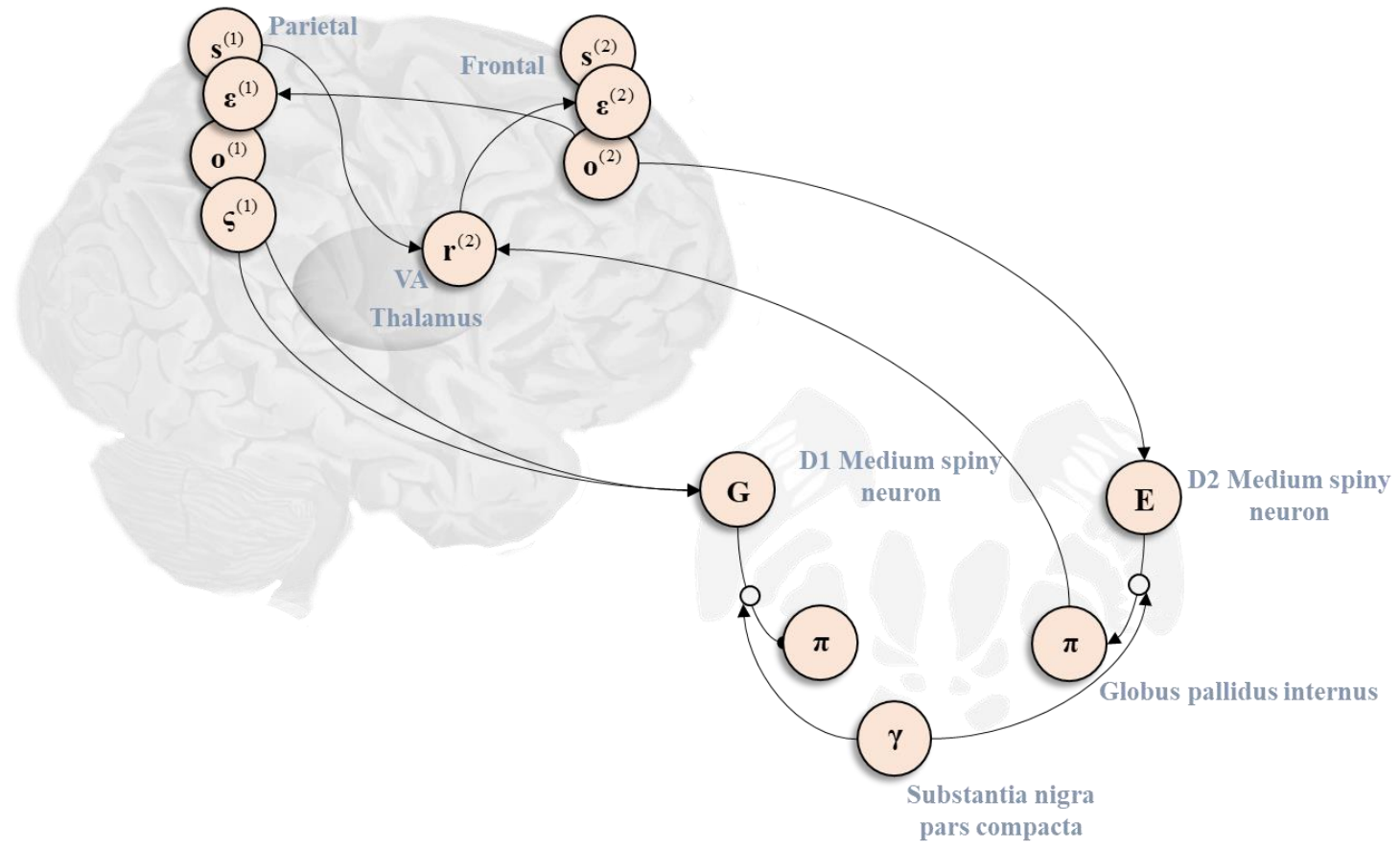


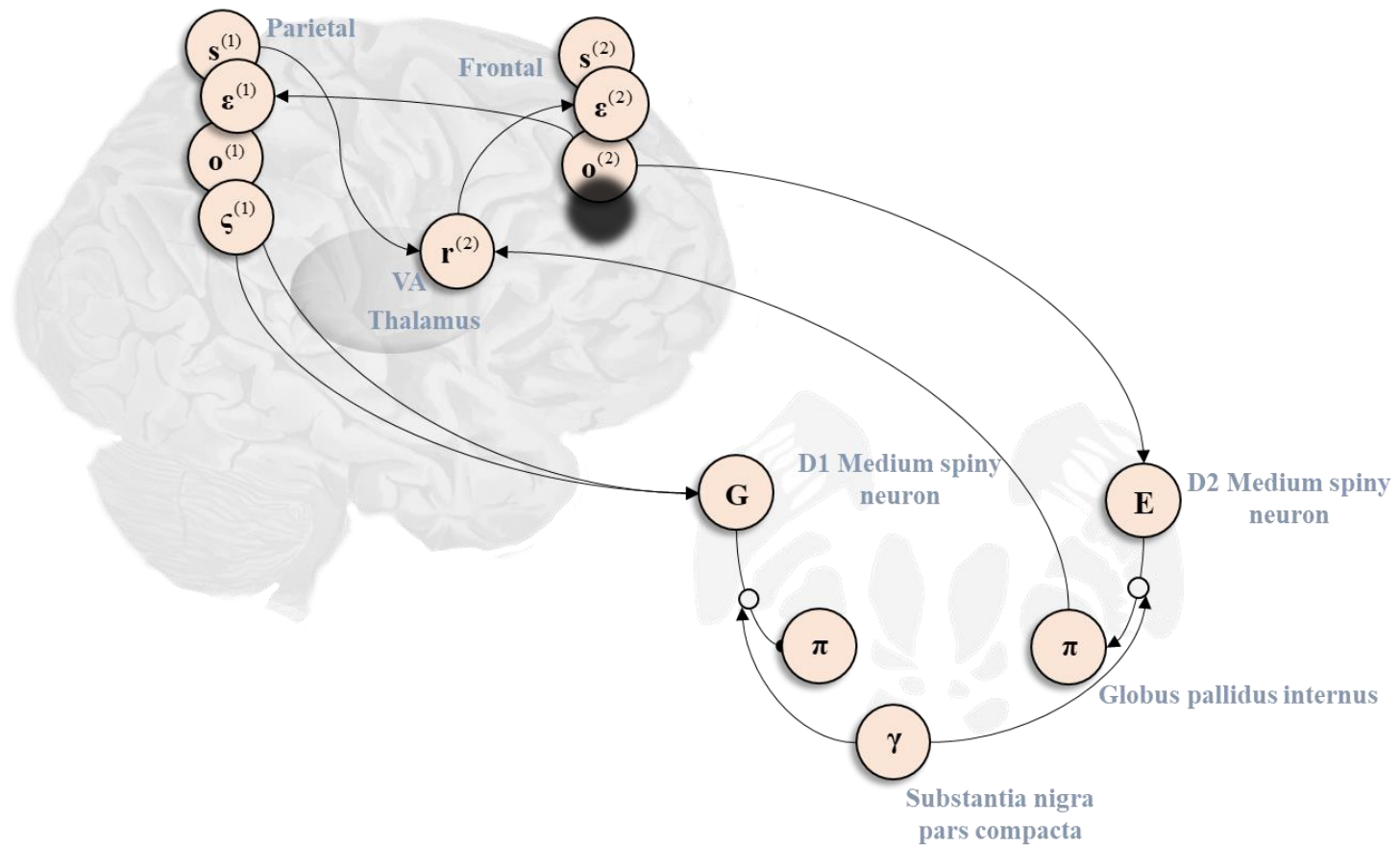
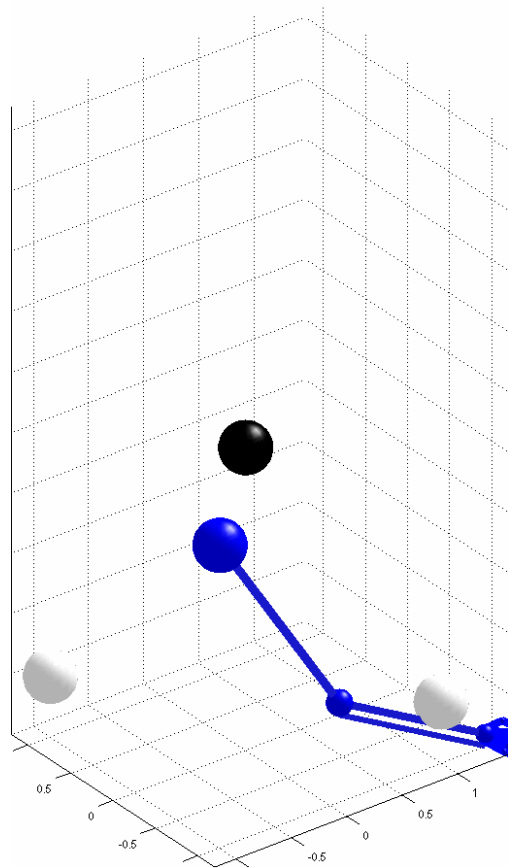


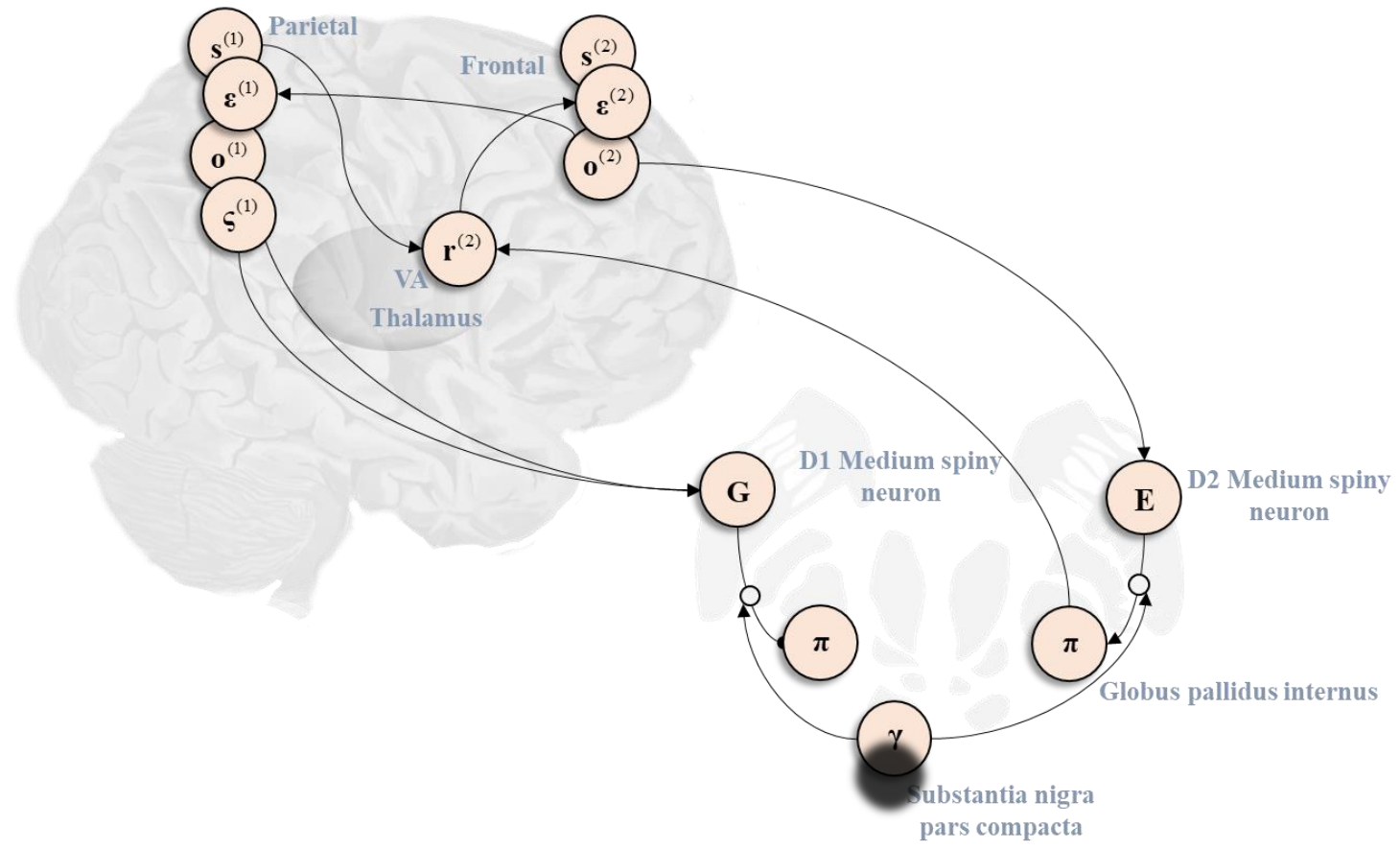
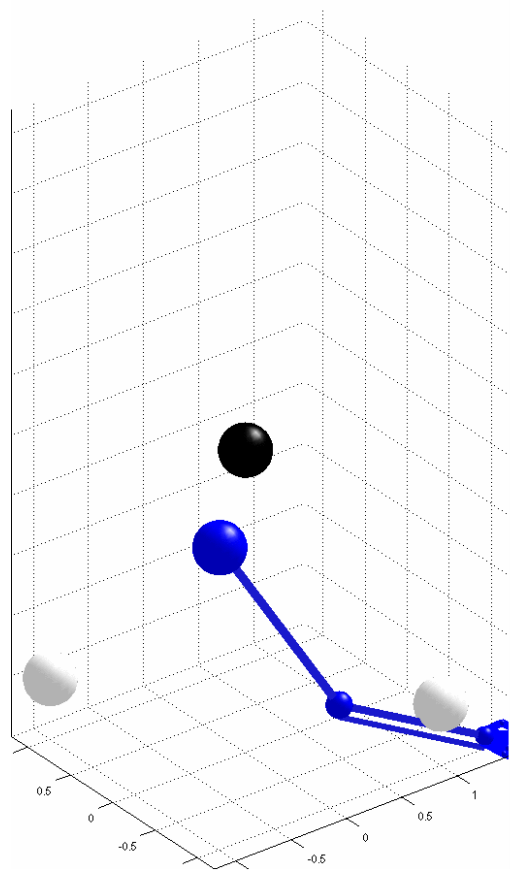




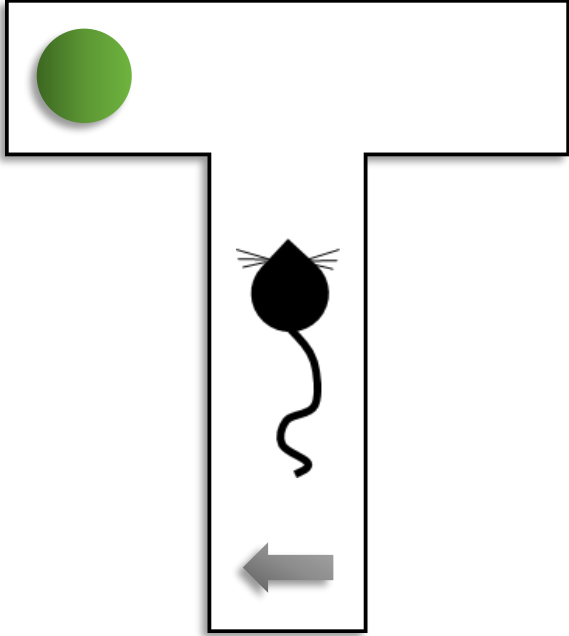


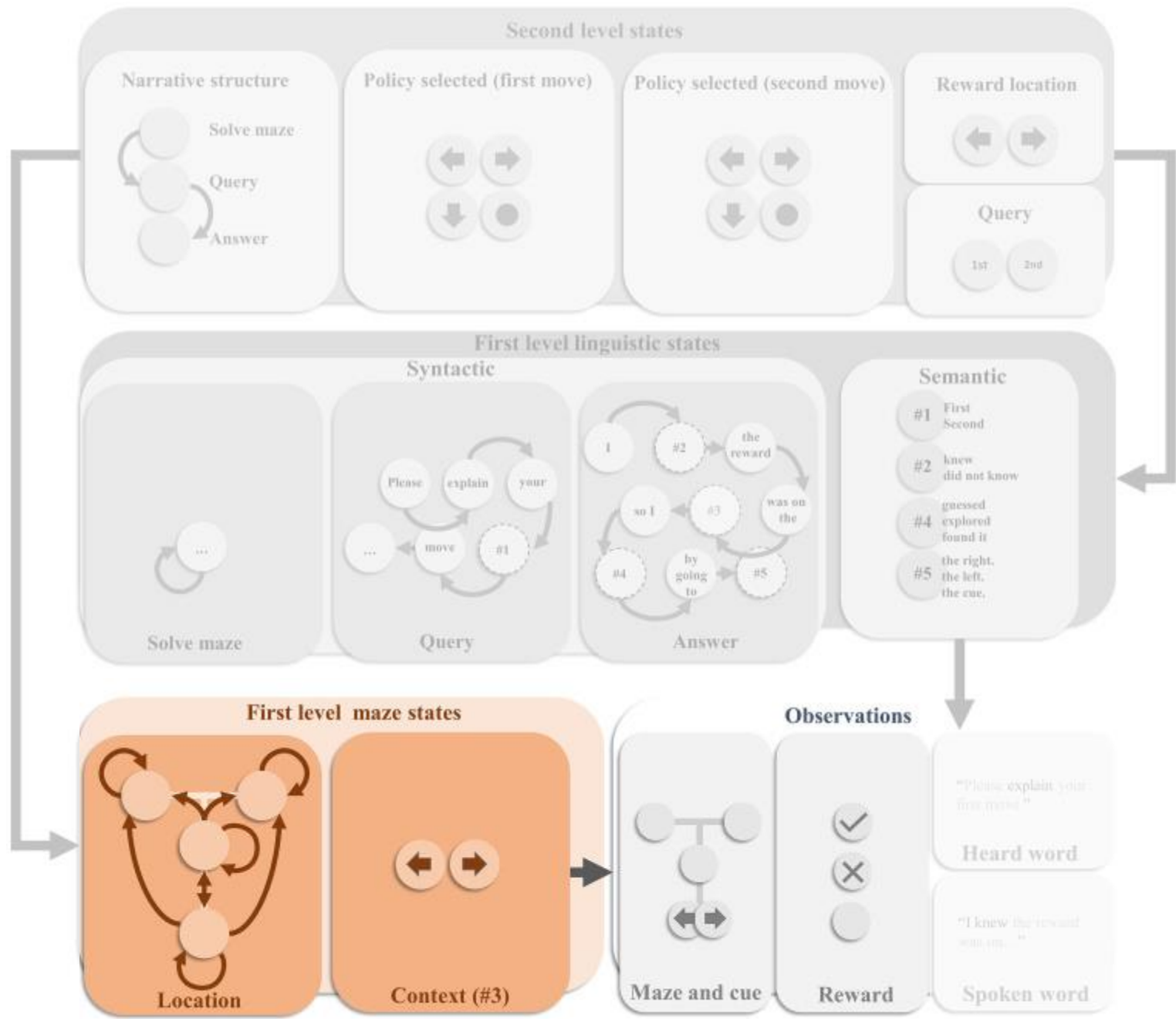


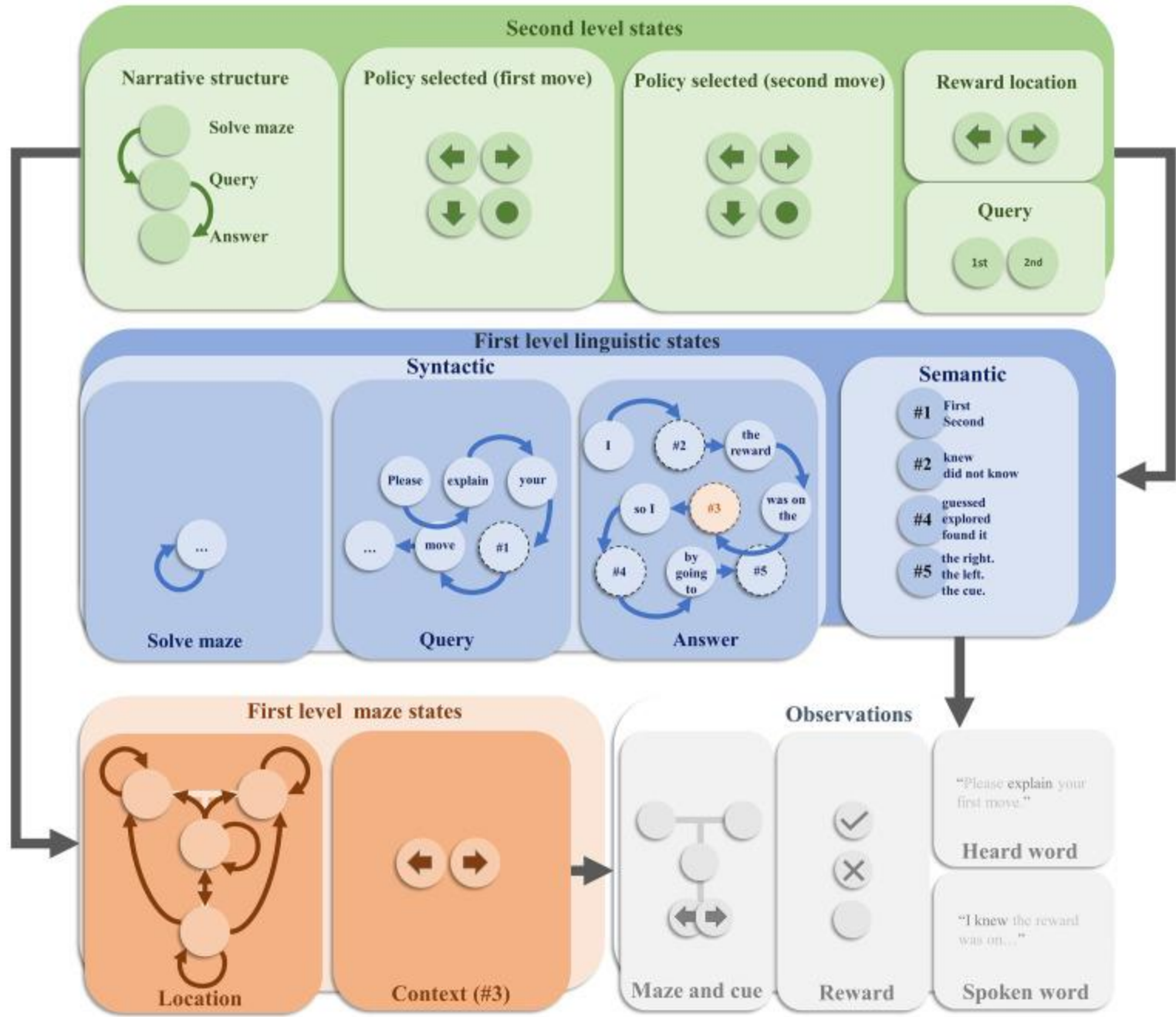










Navigating a T-maze

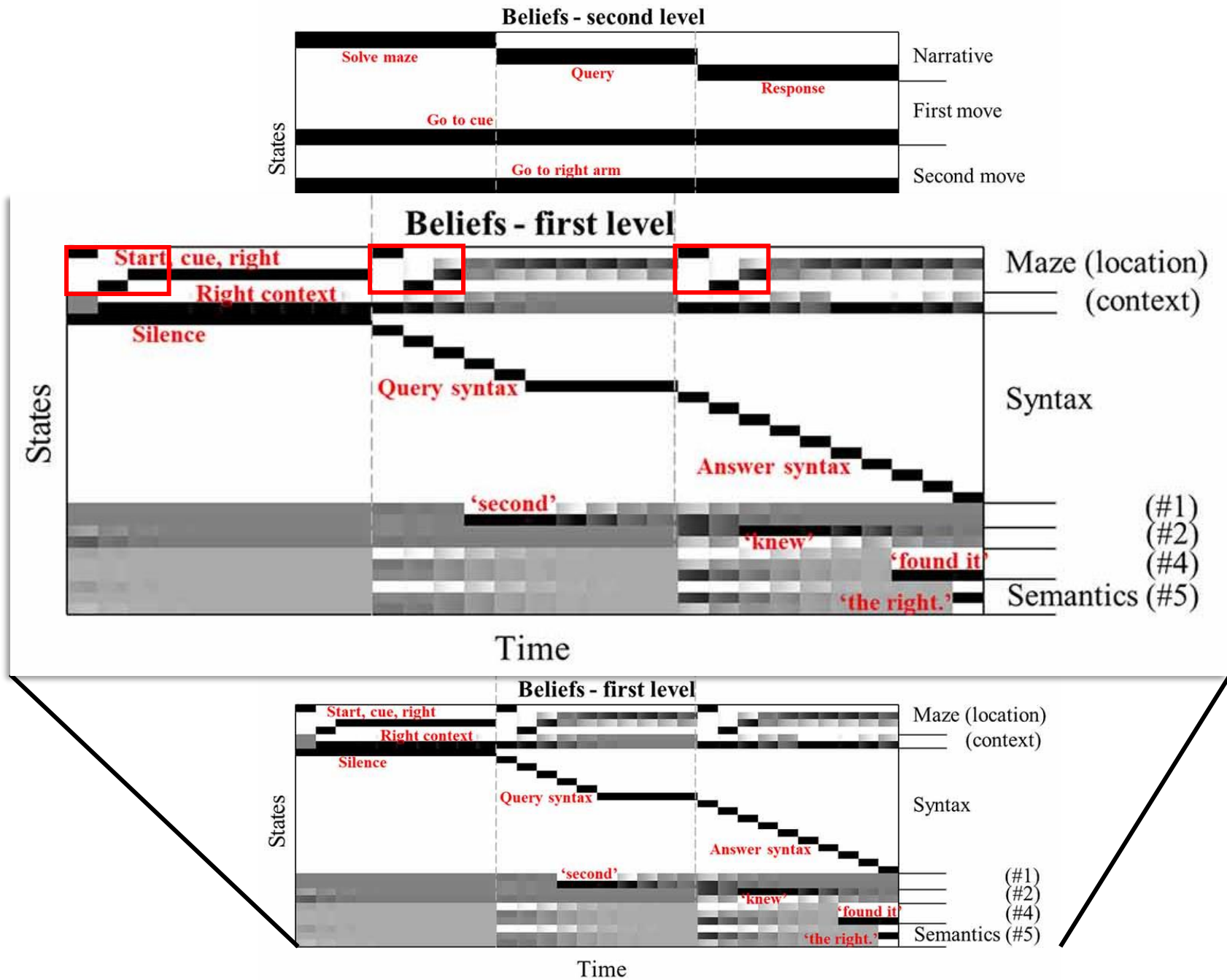









-  Inferred location
-  True location
-  Inferred context

-  Inferred location
-  True location
-  Inferred context



-  Inferred location
-  True location
-  Inferred context

Confabulation



Active Inference

Self-evidencing

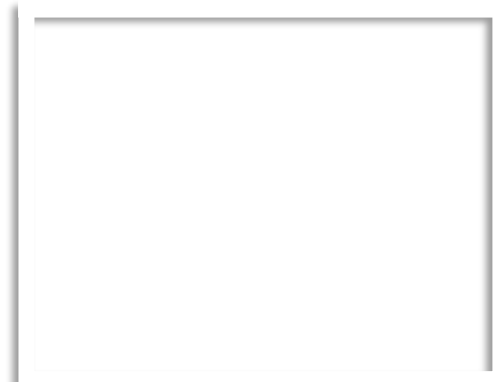
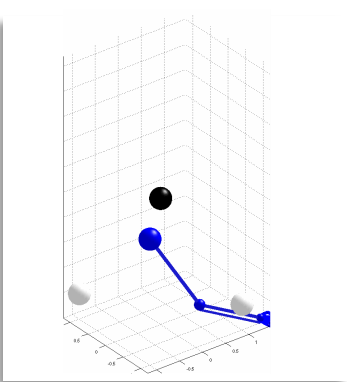
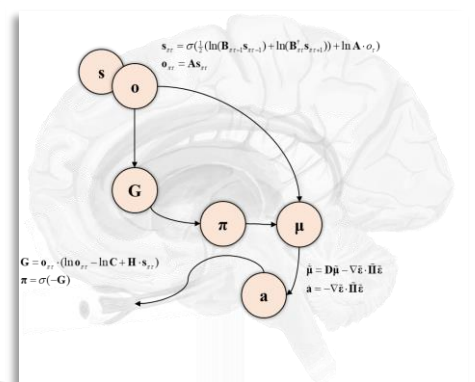
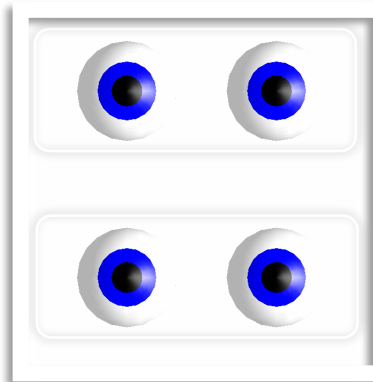
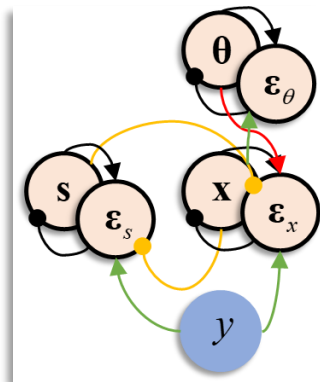
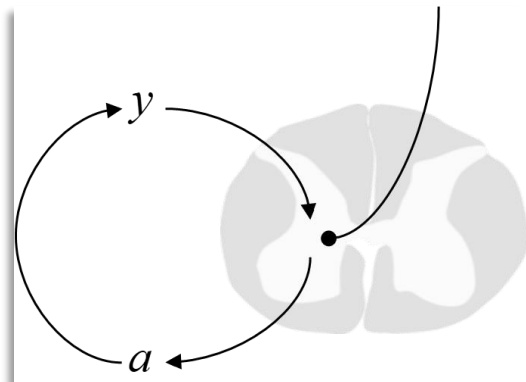
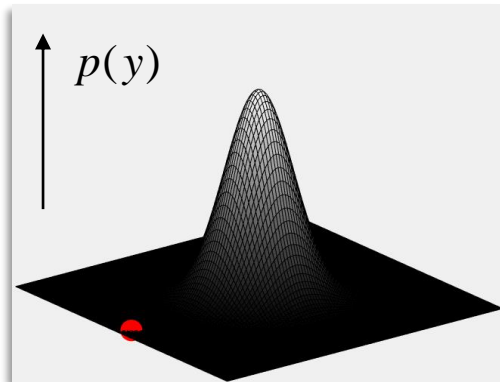
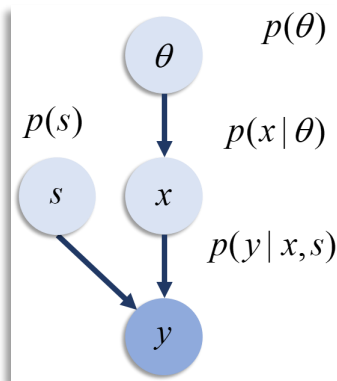
Message passing

Discrete time (planning)

Continuous time (movement)

Hierarchical models

Summary





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And many others



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